

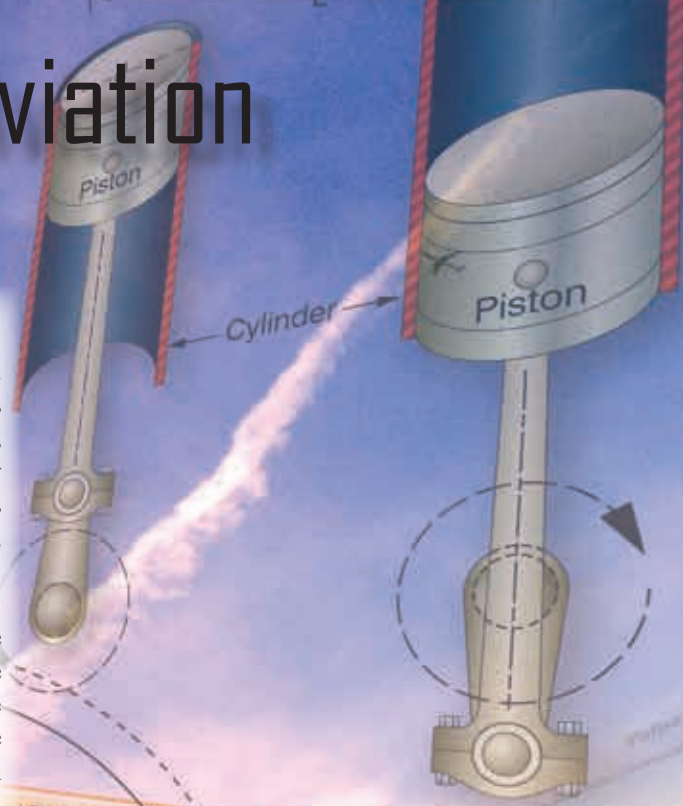
Chapter 3

# Mathematics in Aviation Maintenance

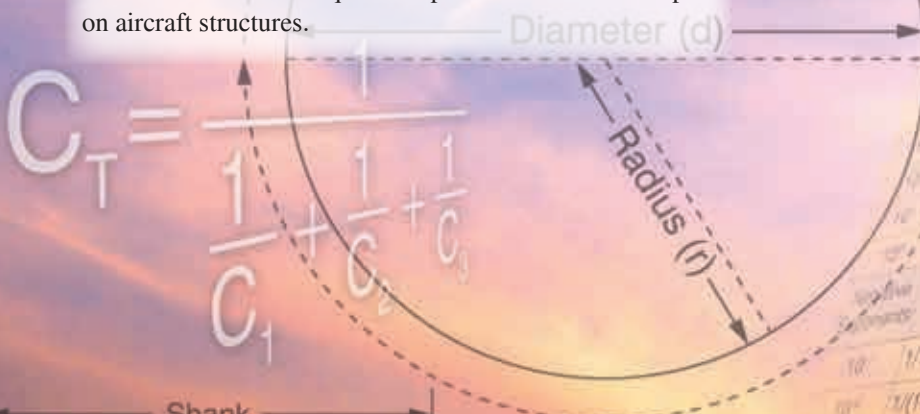
## Introduction

Mathematics is woven into many areas of everyday life. Performing mathematical calculations with success requires an understanding of the correct methods, procedures, practice, and review of these principles. Mathematics may be thought of as a set of tools. The aviation mechanic needs these tools to successfully complete the maintenance, repair, installation, or certification of aircraft equipment.

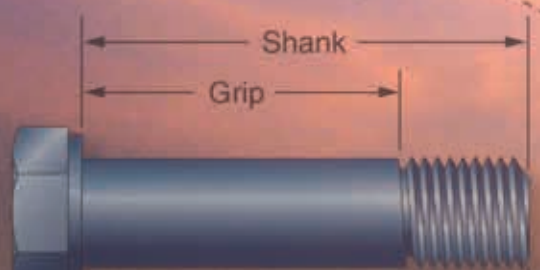
Many examples of using mathematical principles by the aviation mechanic are available. Tolerances in turbine engine components are critical, making it necessary to measure within a ten-thousandth of an inch. Because of these close tolerances, it is important that the aviation mechanic can make accurate measurements and mathematical calculations. An aviation mechanic working on aircraft fuel systems also uses mathematical principles to calculate volumes and capacities of fuel tanks. The use of fractions and surface area calculations are required to perform sheet metal repair on aircraft structures.



10 <sup>0</sup>	1	1,000,000
10 <sup>1</sup>	10	100,000
10 <sup>2</sup>	100	10,000
10 <sup>3</sup>	1,000	1,000
10 <sup>4</sup>	10,000	100
10 <sup>5</sup>	100,000	10
10 <sup>6</sup>	1,000,000	1
10 <sup>-1</sup>	1/10	1/10=0.1
10 <sup>-2</sup>	1/(10 x 10)	1/100=0.01
10 <sup>-3</sup>	1/(10 x 10 x 10)	1/1,000=0.001
10 <sup>-4</sup>	1/(10 x 10 x 10 x 10)	1/10,000=0.0001
10 <sup>-5</sup>	1/(10 x 10 x 10 x 10 x 10)	1/100,000=0.00001
10 <sup>-6</sup>	1/(10 x 10 x 10 x 10 x 10 x 10)	1/1,000,000=0.000001



$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$



## Whole Numbers

Whole numbers are the numbers 0, 1, 2, 3, 4, 5, and so on. Whole numbers can be thought of as counting numbers.

### Addition of Whole Numbers

Addition is the process where the value of one number is added to the value of another. The result is called the sum. When working with whole numbers, it is important to understand the principle of the place value. The place value in a whole number is the value of the position of each individual digit within the entire number. For example, in the number 512, the 5 is in the hundreds column, the 1 is in the tens column, and the 2 is in the ones column. Examples of place values of three whole numbers are shown in *Figure 3-1*.

When adding several whole numbers, such as 4,314, 122, 93,132, and 10, align them into columns according to place value and then add.

$$\begin{array}{r}
 4,314 \\
 122 \\
 93,132 \\
 + 10 \\
 \hline
 97,578
 \end{array}$$

Therefore, 97,578 is the sum of the four whole numbers.

### Subtraction of Whole Numbers

Subtraction is the process where the value of one number is taken from the value of another. The result is called the difference. When subtracting two whole numbers, such as 3,461 from 97,564, align them into columns according to place value and then subtract.

$$\begin{array}{r}
 97,564 \\
 - 3,461 \\
 \hline
 94,103
 \end{array}$$

The difference of the two whole numbers is 94,103.

	Place Value				
	Ten Thousands	Thousands	Hundreds	Tens	Ones
35 shown as				3	5
269 shown as			2	6	9
12,749 shown as	1	2	7	4	9

Figure 3-1. Example of place values of whole numbers.

## Multiplication of Whole Numbers

Multiplication is the process of repeated addition. For example,  $4 \times 3$  is the same as  $4 + 4 + 4$ . The result is called the product.

Example: How many hydraulic system filters do you have if there are 35 cartons in the supply room and each carton contains 18 filters?

$$\begin{array}{r}
 18 \\
 \times 35 \\
 \hline
 90 \\
 54 \\
 \hline
 630
 \end{array}$$

Therefore, there are 630 filters in the supply room.

## Division of Whole Numbers

Division is the process of finding how many times one number (called the divisor) is contained in another number (called the dividend). The result is the quotient, and any amount left over is called the remainder.

$$\begin{array}{r}
 \text{quotient} \\
 \text{divisor} \overline{) \text{dividend}}
 \end{array}$$

Example: 218 landing gear bolts need to be divided between 7 aircraft. How many bolts will each aircraft receive?

$$\begin{array}{r}
 31 \\
 7 \overline{) 218} \\
 - 21 \\
 \hline
 8 \\
 - 7 \\
 \hline
 1
 \end{array}$$

In this case, there are 31 bolts for each of the seven aircraft with one extra remaining.

## Fractions

A fraction is a number written in the form  $\frac{N}{D}$  where N is called the numerator and D is called the denominator. The fraction bar between the numerator and denominator shows that division is taking place.

Some examples of fractions are:  $\frac{17}{18}$ ,  $\frac{2}{3}$ ,  $\frac{5}{8}$

The denominator of a fraction cannot be a zero. For example, the fraction  $\frac{2}{0}$  is not allowed, because dividing by zero is undefined.

An improper fraction is a fraction in which the numerator is equal to or larger than the denominator. For example,  $\frac{4}{4}$  or  $\frac{15}{8}$  are examples of improper fractions.

### Finding the Least Common Denominator

To add or subtract fractions, they must have a common denominator. In math, the least common denominator (LCD) is generally used. One way to find the LCD is to list the multiples of each denominator and then choose the smallest number that they all have in common (can be divided by).

Example: Add  $\frac{1}{5} + \frac{1}{10}$  by finding the LCD.

Multiples of 5 are: 5, 10, 15, 20, 25, and so on. Multiples of 10 are: 10, 20, 30, 40, and so on. Notice that 10, 20, and 30 are in both lists, but 10 is the smallest or LCD. The advantage of finding the LCD is that the final answer should be in the simplest form.

A common denominator can also be found for any group of fractions by multiplying all the denominators together. This number is not always the LCD, but it can still be used to add or subtract fractions.

Example: Add  $\frac{2}{3} + \frac{3}{5} + \frac{4}{7}$  by finding a common denominator.

A common denominator can be found by multiplying the denominators  $3 \times 5 \times 7$  to get 105.

$$\left(\frac{2}{3} + \frac{3}{5} + \frac{4}{7}\right) = \left(\frac{70}{105} + \frac{63}{105} + \frac{60}{105}\right) = \frac{193}{105} = 1\frac{88}{105}$$

### Addition of Fractions

In order to add fractions, the denominators must be the same number. This is referred to as having “common denominators.”

Example: Add  $\frac{1}{7}$  to  $\frac{3}{7}$

$$\frac{1}{7} + \frac{3}{7} = \frac{1+3}{7} = \frac{4}{7}$$

If the fractions do not have the same denominator, then one or all the denominators must be changed so that every fraction has a common denominator.

Example: Find the total thickness of a panel made from  $\frac{3}{32}$ -inch thick aluminum, that has a  $\frac{1}{64}$ -inch thick paint coating. To add these fractions, determine a common denominator. The LCD for this example is 64, so only the first fraction must be changed since the denominator of the second fraction is already in 64ths.

$$\left(\frac{3}{32} + \frac{1}{64}\right) = \left(\frac{3 \times 2}{32 \times 2} + \frac{1}{64}\right) = \left(\frac{6}{64} + \frac{1}{64}\right) = \left(\frac{6+1}{64}\right) = \frac{7}{64}$$

Therefore,  $\frac{7}{64}$  is the total thickness.

### Subtraction of Fractions

To subtract fractions, they must have a common denominator.

Example: Subtract  $\frac{2}{17}$  from  $\frac{10}{17}$

$$\frac{10}{17} - \frac{2}{17} = \frac{10-2}{17} = \frac{8}{17}$$

If the fractions do not have the same denominator, then one or all the denominators must be changed so that every fraction has a common denominator.

Example: The tolerance for rigging the aileron droop of an airplane is  $\frac{7}{8}$  inch  $\pm$   $\frac{1}{5}$  inch. What is the minimum droop to which the aileron can be rigged? To subtract these fractions, first change both to common denominators. The common denominator in this example is 40. Change both fractions to  $\frac{1}{40}$ , as shown, then subtract.

$$\left(\frac{7}{8} - \frac{1}{5}\right) = \left(\frac{7 \times 5}{8 \times 5} - \frac{1 \times 8}{5 \times 8}\right) = \left(\frac{35}{40} - \frac{8}{40}\right) = \left(\frac{35-8}{40}\right) = \frac{27}{40}$$

Therefore,  $\frac{27}{40}$  is the minimum droop.

### Multiplication of Fractions

Multiplication of fractions does not require a common denominator. To multiply fractions, first multiply the numerators. Then, multiply the denominators.

Example:

$$\frac{3}{5} \times \frac{7}{8} \times \frac{1}{2} = \frac{3 \times 7 \times 1}{5 \times 8 \times 2} = \frac{21}{80}$$

The use of cancellation when multiplying fractions is a helpful technique. Cancellation divides out or cancels all common factors that exist between the numerators and denominators. When all common factors are cancelled before the multiplication, the final product is in the simplest form.

Example:

$$\left(\frac{14}{15} \times \frac{3}{7}\right) = \left(\frac{\cancel{14}}{15} \times \frac{\cancel{3}}{\cancel{7}}\right) = \left(\frac{2 \times 1}{5 \times 1}\right) = \frac{2}{5}$$

## Division of Fractions

Division of fractions does not require a common denominator. To divide fractions, first change the division symbol to multiplication. Next, invert the second fraction. Then, multiply the fractions.

Example: Divide  $\frac{7}{8}$  by  $\frac{4}{3}$

$$\left(\frac{7}{8} \div \frac{4}{3}\right) = \left(\frac{7}{8} \times \frac{3}{4}\right) = \left(\frac{7 \times 3}{8 \times 4}\right) = \frac{21}{32}$$

Example: In *Figure 3-2*, the center of the hole is in the center of the plate. Find the distance that the center of the hole is from the edges of the plate. To find the answer, the length and width of the plate should each be divided in half. First, change the mixed numbers to improper fractions:

$$5 \frac{7}{16} \text{ inches} = \frac{87}{16} \text{ inches}$$
$$3 \frac{5}{8} \text{ inches} = \frac{29}{8} \text{ inches}$$

Then, divide each improper fraction by 2 to find the center of the plate.

$$\frac{87}{16} \div \frac{2}{1} = \frac{87}{16} \times \frac{1}{2} = \frac{87}{32} \text{ inches}$$

$$\frac{29}{8} \div \frac{2}{1} = \frac{29}{8} \times \frac{1}{2} = \frac{29}{16} \text{ inches}$$

Finally, convert each improper fraction to a mixed number:

$$\frac{87}{32} = 87 \div 32 = 2 \frac{23}{32} \text{ inches}$$

$$\frac{29}{16} = 29 \div 16 = 1 \frac{13}{16} \text{ inches}$$

Therefore, the distance to the center of the hole from each of the plate edges is  $2 \frac{23}{32}$  inches and  $1 \frac{13}{16}$  inches.

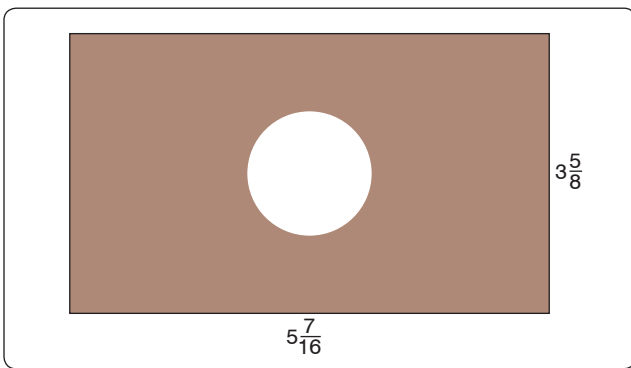


Figure 3-2. Center hole of the plate.

## Reducing Fractions

A fraction needs to be reduced when it is not in the simplest form or “lowest terms.” Lowest term means that the numerator and denominator do not have any factors in common. That is, they cannot be divided by the same number (or factor). To reduce a fraction, determine what the common factor(s) are and divide these out of the numerator and denominator. For example, when both the numerator and denominator are even numbers, they can both be divided by 2.

Example: The total travel of a jackscrew is  $\frac{13}{16}$  inch. If the travel in one direction from the neutral position is  $\frac{7}{16}$  inch, what is the travel in the opposite direction?

$$\frac{13}{16} - \frac{7}{16} = \frac{13-7}{16} = \frac{6}{16}$$

The fraction  $\frac{6}{16}$  is not in lowest terms because the numerator (6) and the denominator (16) have a common factor of 2. To reduce  $\frac{6}{16}$ , divide the numerator and the denominator by 2. The final reduced fraction is  $\frac{3}{8}$  as shown below.

$$\frac{6}{16} = \frac{6 \div 2}{16 \div 2} = \frac{3}{8}$$

Therefore, the travel in the opposite direction is  $\frac{3}{8}$  inch.

## Mixed Numbers

A mixed number is a combination of a whole number and a fraction.

### Addition of Mixed Numbers

To add mixed numbers, add the whole numbers together. Then add the fractions together by finding a common denominator. The final step is to add the sum of the whole numbers to the sum of the fractions for the final answer.

Example: The cargo area behind the rear seat of a small airplane can handle solids that are  $4 \frac{3}{4}$  feet long. If the rear seats are removed, then  $2 \frac{1}{3}$  feet is added to the cargo area. What is the total length of the cargo area when the rear seats are removed?

$$4 \frac{3}{4} + 2 \frac{1}{3} = (4 + 2) + \left(\frac{3}{4} + \frac{1}{3}\right) = 6 + \left(\frac{9}{12} + \frac{4}{12}\right) = 6 \frac{13}{12} = 7 \frac{1}{2} \text{ feet of cargo room}$$

### Subtraction of Mixed Numbers

To subtract mixed numbers, find a common denominator for the fractions. Subtract the fractions from each other. If

may be necessary to borrow from the larger whole number when subtracting the fractions. Subtract the whole numbers from each other. The final step is to combine the final whole number with the final fraction.

Example: What is the length of the grip of the bolt shown in *Figure 3-3*? The overall length of the bolt is  $3 \frac{1}{2}$  inches, the shank length is  $3 \frac{1}{8}$  inches, and the threaded portion is  $1 \frac{5}{16}$  inches long. To find the grip, subtract the length of the threaded portion from the length of the shank.

$$3 \frac{1}{8} \text{ inches} - 1 \frac{5}{16} \text{ inches} = \text{grip length}$$

To subtract, start with the fractions. Borrowing is necessary because  $\frac{1}{8}$  is larger than  $\frac{5}{16}$  (or  $\frac{2}{16}$ ). From the whole number 3, borrow 1, which is actually  $\frac{16}{16}$ . After borrowing, the first mixed number is now  $2 \frac{18}{16}$ . This is because,  $3 \frac{1}{8} = 3 \frac{2}{16} = 2 + 1 + \frac{2}{16} = 2 + \frac{16}{16} + \frac{2}{16} = 2 \frac{18}{16}$ .

$$3 \frac{1}{8} - 1 \frac{5}{16} = 3 \frac{2}{16} - 1 \frac{5}{16} = 2 \frac{18}{16} - 1 \frac{5}{16} = 1 \frac{13}{16}$$

Therefore, the grip length of the bolt is  $1 \frac{13}{16}$  inches.

(Note: The value for the overall length of the bolt was given in the example, but it was not needed to solve the problem. This type of information is sometimes referred to as a “distracter,” because it distracts from the information needed to solve the problem.)

## The Decimal Number System

### Origin and Definition

The number system that we use every day is called the decimal system. The prefix in the word decimal, dec, is a Latin root for the word “ten.” The decimal system probably originated from the fact that we have ten fingers (or digits). The decimal system has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The decimal system is a base 10 system and has been in use for over 5,000 years. A decimal is a number with a decimal point. For example, 0.515, 0.10, and 462.625 are all decimal numbers. Like whole numbers, decimal numbers also have

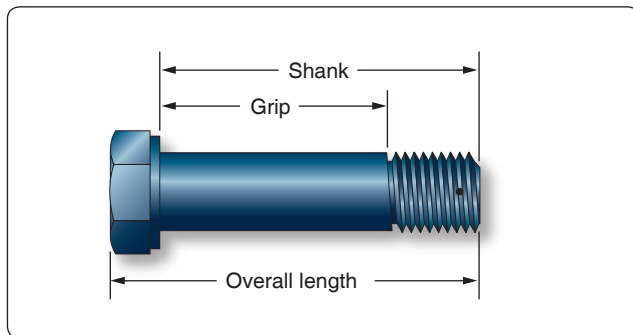


Figure 3-3. Bolt dimensions.

Place Value											
	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
1,623,051	1	6	2	3	0	5	1				
0.0531							0	0	5	3	1
32.4						3	2	4			

Figure 3-4. Place values.

place value. The place values are based on powers of 10, as shown in *Figure 3-4*.

### Addition of Decimal Numbers

To add decimal numbers, they must first be arranged so that the decimal points are aligned vertically and according to place value. That is, adding tenths with tenths, ones with ones, hundreds with hundreds, and so forth.

Example: Find the total resistance for the circuit diagram shown in *Figure 3-5*. The total resistance of a series circuit is equal to the sum of the individual resistances. To find the total resistance,  $R_T$ , the individual resistances are added together.

$$R_T = 2.34 + 37.5 + 0.09$$

Arrange the resistance values in a vertical column so that the decimal points are aligned and then add.

$$\begin{array}{r} 2.34 \\ 37.5 \\ + 0.09 \\ \hline 39.93 \end{array}$$

Therefore, the total resistance,  $R_T = 39.93$  ohms.

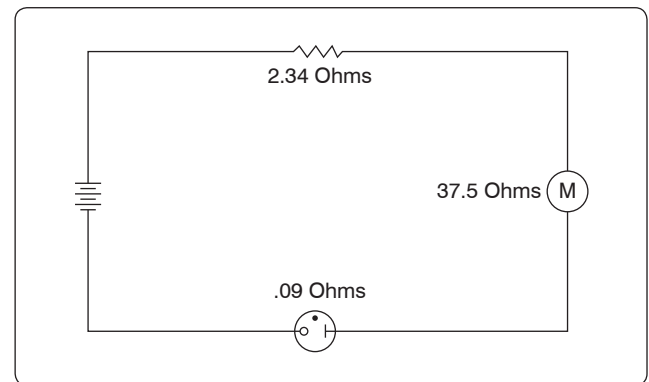


Figure 3-5. Circuit diagram.

## Subtraction of Decimal Numbers

To subtract decimal numbers, they must first be arranged so that the decimal points are aligned vertically and according to place value. That is, subtracting tenths from tenths, ones from ones, hundreds from hundreds, and so forth.

Example: A series circuit containing two resistors has a total resistance ( $R_T$ ) of 37.272 ohms. One of the resistors ( $R_1$ ) has a value of 14.88 ohms. What is the value of the other resistor ( $R_2$ )?

$$R_2 = R_T - R_1 = 37.272 - 14.88$$

Arrange the decimal numbers in a vertical column so that the decimal points are aligned and then subtract.

$$\begin{array}{r} 37.272 \\ -14.88 \\ \hline 22.392 \end{array}$$

Therefore, the second resistor,  $R_2 = 22.392$  ohms.

## Multiplication of Decimal Numbers

To multiply decimal numbers, vertical alignment of the decimal point is not required. Instead, align the numbers to the right in the same way that whole numbers are multiplied (with no regard to the decimal points or place values) and then multiply. The last step is to place the decimal point in the correct place in the answer. To do this, count the number of decimal places in each of the numbers, add the total, and then assign that number of decimal places to the result.

Example: To multiply  $0.2 \times 6.03$ , arrange the numbers vertically and align them to the right. Multiply the numbers, ignoring the decimal points for now.

$$\begin{array}{r} 6.03 \\ \times 0.2 \\ \hline 1206 \end{array} \quad (\text{ignore the decimal points, for now})$$

After multiplying the numbers, count the total number of decimal places in both numbers. For this example, 6.03 has 2 decimal places and 0.2 has 1 decimal place. Together there are a total of 3 decimal places. The decimal point for the answer is placed 3 decimal places from the right. Therefore, the answer is 1.206.

$$\begin{array}{r} 6.03 \quad \leftarrow 2 \text{ decimal places} \\ \times 0.2 \quad \leftarrow 1 \text{ decimal place} \\ \hline 1.206 \quad \leftarrow 3 \text{ decimal places} \end{array}$$

Example: Using the formula  $\text{watts} = \text{amperes} \times \text{voltage}$ , what is the wattage of an electric drill that uses 9.45 amperes from a 120-volt source? Align the numbers to the right and multiply.

After multiplying the numbers, count the total number of decimal places in both numbers. For this example, 9.45 has 2 decimal places and 120 has no decimal place. Together there are 2 decimal places. The decimal point for the answer is placed 2 decimal places from the right. Therefore, the answer is 1,134.00 watts, or simplified to 1,134 watts.

$$\begin{array}{r} 9.45 \quad \leftarrow 2 \text{ decimal places} \\ \times 120 \quad \leftarrow \text{no decimal place} \\ \hline 000 \\ 1890 \\ + 945 \\ \hline 1,134.00 \quad \leftarrow 2 \text{ decimal places} \end{array}$$

## Division of Decimal Numbers

Division of decimal numbers is performed the same way as whole numbers, unless the divisor is a decimal.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

When the divisor is a decimal, it must be changed to a whole number before dividing. To do this, move the decimal in the divisor to the right until there are no decimal places. At the same time, move the decimal point in the dividend to the right the same number of places. Then divide. The decimal in the quotient is placed directly above the decimal in the dividend.

Example: Divide 0.144 by 0.12

$$0.12 \overline{) 0.144} = 12. \overline{) 14.4}$$

$$\begin{array}{r} 1.2 \\ 12 \overline{) 14.4} \\ - 12 \\ \hline 24 \\ - 24 \\ \hline 0 \end{array}$$

Move the decimal in the divisor (0.12) two places to the right. The result is 12.0. Next, move the decimal in the dividend (0.144) two places to the right. The result is 14.4. Now divide. The result is 1.2.

Example: The wing area of an airplane is 262.6 square feet and its span is 40.4 feet. Find the mean chord of its wing using the formula:  $\text{area} \div \text{span} = \text{mean chord}$ .

$$\begin{array}{r}
 40.4 \overline{) 262.6} = 404. \overline{) 2626.0} \\
 \underline{- 2424} \\
 2020 \\
 \underline{- 2020} \\
 0
 \end{array}$$

Move the decimal in the divisor (40.4) one place to the right. Next, move the decimal in the dividend (262.6) one place to the right. Then divide. The mean chord length is 6.5 feet.

### Rounding Off Decimal Numbers

Occasionally, it is necessary to round off a decimal number to some value that is practical to use. For example, a measurement is calculated to be 29.4948 inches. To use this measurement, we can use the process of “rounding off.” A decimal is “rounded off” by keeping the digits for a certain number of places and discarding the rest. The degree of accuracy desired determines the number of digits to be retained. When the digit immediately to the right of the last retained digit is 5 or greater, round up by 1. When the digit immediately to the right of the last retained digit is less than 5, leave the last retained digit unchanged.

Example: An actuator shaft is 2.1938 inches in diameter. Round to the nearest tenth.

The digit in the tenths column is a 1. The digit to the right of the 1 is a 9. Since 9 is greater than or equal to 5, “round up” the 1 to a 2. Therefore, 2.1938 rounded to the nearest tenth is 2.2.

Example: The outside diameter of a bearing is 3.1648 centimeters. Round to the nearest hundredth.

The digit in the hundredths column is a 6. The digit to the right of the 6 is a 4. Since 4 is less than 5, do not round up the 6. Therefore, 3.1648 to the nearest hundredth is 3.16.

Example: The length of a bushing is 3.7487 feet. Round to the nearest thousandth.

The digit in the thousandths column is an 8. The digit to the right of the 8 is a 7. Since 7 is greater than or equal to 5, “round up” the 8 to a 9. Therefore, 3.7487 to the nearest thousandth is 3.749.

### Converting Decimal Numbers to Fractions

To change a decimal number to a fraction, “read” the decimal out loud, and then write it into a fraction just as it is read as shown below.

Example: One oversized rivet has a diameter of 0.52 inches. Convert 0.52 to a fraction. The decimal 0.52 is read as “fifty-two hundredths.”

$$0.52 = \frac{52}{100} \leftarrow \begin{array}{l} \text{“fifty-two”} \\ \text{“hundredths”} \end{array}$$

In the above fraction of  $\frac{52}{100}$ , we can divide 4 into each number resulting in a fraction of  $\frac{13}{25}$ .

A dimension often appears in a maintenance manual or on a blueprint as a decimal instead of a fraction. To use the dimension, it may need to be converted to a fraction. An aviation mechanic frequently uses a steel rule that is calibrated in units of  $\frac{1}{64}$  of an inch. To change a decimal to the nearest equivalent common fraction, multiply the decimal by 64. The product of the decimal and 64 is the numerator of the fraction and 64 is the denominator. Reduce the fraction, if needed.

Example: The width of a hex head bolt is 0.3123 inches. Convert the decimal 0.3123 to a common fraction to decide which socket would be the best fit for the bolt head. First, multiply the 0.3123 decimal by 64:

$$0.3123 \times 64 = 19.9872$$

Next, round the product to the nearest whole number:  $19.98722 \approx 20$ .

Use this whole number (20) as the numerator and 64 as the denominator:  $\frac{20}{64}$ .

Now, reduce  $\frac{20}{64}$  to  $\frac{5}{16}$  as 4 is common to both the numerator and denominator. Therefore, the correct socket would be the  $\frac{5}{16}$  inch socket ( $\frac{20}{64}$  reduced).

Example: When accurate holes of uniform diameter are required for aircraft structures, they are first drilled approximately  $\frac{1}{64}$  inch undersized and then reamed to the final desired diameter. What size drill bit should be selected for the undersized hole if the final hole is reamed to a diameter of 0.763 inches? First, multiply the 0.763 decimal by 64.

$$0.763 \times 64 = 48.832$$

Next, round the product to the nearest whole number:  $48.832 \approx 49$ .

Use this number (49) as the numerator and 64 as the denominator:  $\frac{49}{64}$  is the closest fraction to the final reaming diameter of 0.763 inches. To determine the drill size for the initial undersized hole, subtract  $\frac{1}{64}$  inch from the finished hole size.

$$\frac{49}{64} - \frac{1}{64} = \frac{48}{64} = \frac{3}{4}$$

Therefore, a  $\frac{3}{4}$ -inch drill bit should be used for the initial undersized holes.

### Converting Fractions to Decimals

To convert any fraction to a decimal, simply divide the top number (numerator) by the bottom number (denominator). Every fraction has an approximate decimal equivalent.

Example:

$$\frac{1}{2} = 1 \div 2 = 2 \overline{)1.0} \begin{array}{r} 0.5 \\ - 1.0 \\ \hline 0 \end{array} \quad \text{Therefore, } \frac{1}{2} = 0.5$$

$$\frac{3}{8} = 3 \div 8 = 8 \overline{)3.000} \begin{array}{r} 0.375 \\ - 24 \\ \hline 60 \\ - 56 \\ \hline 40 \\ - 40 \\ \hline 0 \end{array} \quad \text{Therefore, } \frac{3}{8} = 0.375$$

Calculator tip: numerator (top number)  $\div$  denominator (bottom number) = the decimal equivalent of the fraction.

Some fractions when converted to decimals produce a repeating decimal.

Example:

$$\frac{1}{3} = 1 \div 3 = 3 \overline{)1.00} = 0.33 \text{ or } 0.3\overline{3}$$

This decimal can be represented with a bar, or can be rounded. (A bar indicates that the number(s) beneath it are repeated to infinity.)

Other examples of repeating decimals:

$$0.212121\dots = 0.2\overline{1}$$

$$0.6666\dots = 0.6\overline{6} \text{ or } 0.67$$

$$0.254254\dots = 0.25\overline{4}$$

### Decimal Equivalent Chart

Figure 3-6 is a fraction to decimal to millimeter equivalency chart. Measurements starting at  $\frac{1}{64}$  inch and up to 23 inches have been converted to decimal numbers and to millimeters.

### Ratio

A ratio is the comparison of two numbers or quantities. A ratio may be expressed in three ways: as a fraction, with a colon, or with the word “to.” For example, a gear ratio of 5:7 can be expressed as any of the following:

$$\frac{5}{7} \text{ or } 5:7 \text{ or } 5 \text{ to } 7$$

### Aviation Applications

Ratios have widespread application in the field of aviation.

Example: Compression ratio on a reciprocating engine is the ratio of the volume of a cylinder with the piston at the bottom of its stroke to the volume of the cylinder with the piston at the top of its stroke. For example, a typical compression ratio might be 10:1 (or 10 to 1).

Aspect ratio is the ratio of the length (or span) of an airfoil to its width (or chord). A typical aspect ratio for a commercial airliner might be 7:1 (or 7 to 1).

Air-fuel ratio is the ratio of the weight of the air to the weight of fuel in the mixture being fed into the cylinders of a reciprocating engine. For example, a typical air-fuel ratio might be 14.3:1 (or 14.3 to 1).

Glide ratio is the ratio of the forward distance traveled to the vertical distance descended when an aircraft is operating without power. For example, if an aircraft descends 1,000 feet while it travels through the air for two linear miles (10,560 feet), it has a glide ratio of 10,560:1,000 which can be reduced to 10.56:1 (or 10.56 to 1).

Gear ratio is the number of teeth each gear represents when two gears are used in an aircraft component. In Figure 3-7, the pinion gear has 8 teeth and a spur gear has 28 teeth. The gear ratio is 8:28. Using 7 as the LCD, 8:28 becomes 2:7.

Speed ratio is when two gears are used in an aircraft component; the rotational speed of each gear is represented as a speed ratio. As the number of teeth in a gear decreases, the rotational speed of that gear increases, and vice-versa. Therefore, the speed ratio of two gears is the inverse (or opposite) of the gear ratio. If two gears have a gear ratio of 2:9, then their speed ratio is 9:2.

Example: A pinion gear with 10 teeth is driving a spur gear with 40 teeth. The spur gear is rotating at 160 rpm. Calculate the speed of the pinion gear.

$$\frac{\text{Teeth in Pinion Gear}}{\text{Teeth in Spur Gear}} = \frac{\text{Speed of Spur Gear}}{\text{Speed of Pinion Gear}}$$



Fraction	Decimal	MM
1/64	0.015	0.396
1/32	0.031	0.793
3/64	0.046	1.190
1/16	0.062	1.587
5/64	0.078	1.984
3/32	0.093	2.381
7/64	0.109	2.778
1/8	0.125	3.175
9/64	0.140	3.571
5/32	0.156	3.968
11/64	0.171	4.365
3/16	0.187	4.762
13/64	0.203	5.159
7/32	0.218	5.556
15/64	0.234	5.953
1/4	0.25	6.35
17/64	0.265	6.746
9/32	0.281	7.143
19/64	0.296	7.540
5/16	0.312	7.937
21/64	0.328	8.334
11/32	0.343	8.731
23/64	0.359	9.128
3/8	0.375	9.525
25/64	0.390	9.921
13/32	0.406	10.318
27/64	0.421	10.715
7/16	0.437	11.112
29/64	0.453	11.509
15/32	0.468	11.906
31/64	0.484	12.303
1/2	0.5	12.7
33/64	0.515	13.096
17/32	0.531	13.493
35/64	0.546	13.890
39/64	0.562	14.287
37/64	0.578	14.684
19/32	0.593	15.081
39/64	0.609	15.478
5/8	0.625	15.875
41/64	0.640	16.271
21/32	0.656	16.668
43/64	0.671	17.065
11/16	0.687	17.462
45/64	0.703	17.859
23/32	0.718	18.256
47/64	0.734	18.653
3/4	0.75	19.05
49/64	0.765	19.446
25/32	0.781	19.843
51/64	0.796	20.240
13/16	0.812	20.637
53/64	0.828	21.034
27/32	0.843	21.431
55/64	0.859	21.828
7/8	0.875	22.225
57/64	0.890	22.621
29/32	0.906	23.018
59/64	0.921	23.415
15/16	0.937	23.812
61/64	0.953	24.209
31/32	0.968	24.606
63/64	0.984	25.003
1	1	25.4

Fraction	Decimal	MM
1 1/64	1.015	25.796
1 1/32	1.031	26.193
1 3/64	1.046	26.590
1 1/16	1.062	26.987
1 5/64	1.078	27.384
1 3/32	1.093	27.781
1 7/64	1.109	28.178
1 1/8	1.125	28.575
1 9/64	1.140	28.971
1 5/32	1.156	29.368
1 11/64	1.171	29.765
1 3/16	1.187	30.162
1 13/64	1.203	30.559
1 7/32	1.218	30.956
1 15/64	1.234	31.353
1 1/4	1.25	31.75
1 17/64	1.265	32.146
1 9/32	1.281	32.543
1 19/64	1.296	32.940
1 5/16	1.312	33.337
1 21/64	1.328	33.734
1 11/32	1.343	34.131
1 23/64	1.359	34.528
1 3/8	1.375	34.925
1 25/64	1.390	35.321
1 13/32	1.406	35.718
1 27/64	1.421	36.115
1 7/16	1.437	36.512
1 29/64	1.453	36.909
1 15/32	1.468	37.306
1 31/64	1.484	37.703
1 1/2	1.5	38.1
1 33/64	1.515	38.496
1 17/32	1.531	38.893
1 35/64	1.546	39.290
1 9/16	1.562	39.687
1 37/64	1.578	40.084
1 19/32	1.593	40.481
1 39/64	1.609	40.878
1 5/8	1.625	41.275
1 41/64	1.640	41.671
1 21/32	1.656	42.068
1 43/64	1.671	42.465
1 11/16	1.687	42.862
1 45/64	1.703	43.259
1 23/32	1.718	43.656
1 47/64	1.734	44.053
1 3/4	1.75	44.45
1 49/64	1.765	44.846
1 25/32	1.781	45.243
1 51/64	1.796	45.640
1 13/16	1.812	46.037
1 53/64	1.828	46.434
1 27/32	1.843	46.831
1 55/64	1.859	47.228
1 7/8	1.875	47.625
1 57/64	1.890	48.021
1 29/32	1.906	48.418
1 59/64	1.921	48.815
1 15/16	1.937	49.212
1 61/64	1.953	49.609
1 31/32	1.968	50.006
1 63/64	1.984	50.403
2	2	50.8

Fraction	Decimal	MM
2 1/64	2.015	51.196
2 1/32	2.031	51.593
2 3/64	2.046	51.990
2 1/16	2.062	52.387
2 5/64	2.078	52.784
2 3/32	2.093	53.181
2 7/64	2.109	53.578
2 1/8	2.125	53.975
2 9/64	2.140	54.371
2 5/32	2.156	54.768
2 11/64	2.171	55.165
2 3/16	2.187	55.562
2 13/64	2.203	55.959
2 7/32	2.218	56.356
2 15/64	2.234	56.753
2 1/4	2.25	57.15
2 17/64	2.265	57.546
2 9/32	2.281	57.943
2 19/64	2.296	58.340
2 5/16	2.312	58.737
2 21/64	2.328	59.134
2 11/32	2.343	59.531
2 23/64	2.359	59.928
2 3/8	2.375	60.325
2 25/64	2.390	60.721
2 13/32	2.406	61.118
2 27/64	2.421	61.515
2 7/16	2.437	61.912
2 29/64	2.453	62.309
2 15/32	2.468	62.706
2 31/64	2.484	63.103
2 1/2	2.5	63.5
2 33/64	2.515	63.896
2 17/32	2.531	64.293
2 35/64	2.546	64.690
2 9/16	2.562	65.087
2 37/64	2.578	65.484
2 19/32	2.593	65.881
2 39/64	2.609	66.278
2 5/8	2.625	66.675
2 41/64	2.640	67.071
2 21/32	2.656	67.468
2 43/64	2.671	67.865
2 11/16	2.687	68.262
2 45/64	2.703	68.659
2 23/32	2.718	69.056
2 47/64	2.734	69.453
2 3/4	2.75	69.85
2 49/64	2.765	70.246
2 25/32	2.781	70.643
2 51/64	2.796	71.040
2 13/16	2.812	71.437
2 53/64	2.828	71.834
2 27/32	2.843	72.231
2 55/64	2.859	72.628
2 7/8	2.875	73.025
2 57/64	2.890	73.421
2 29/32	2.906	73.818
2 59/64	2.921	74.215
2 15/16	2.937	74.612
2 61/64	2.953	75.009
2 31/32	2.968	75.406
2 63/64	2.984	75.803
3	3	76.2

Figure 3-6. Fractions, decimals, and millimeters.

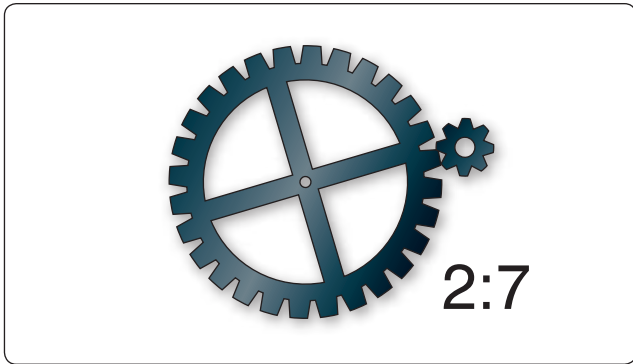


Figure 3-7. Gear ratio.

$$\frac{10 \text{ teeth}}{40 \text{ teeth}} = \frac{160 \text{ rpm}}{S_p \text{ (speed of pinion gear)}}$$

To solve for  $S_p$ , multiply  $40 \times 160$ , then divide by 10. The speed of the pinion gear is 640 rpm.

Example: If the cruising speed of an airplane is 200 knots and its maximum speed is 250 knots, what is the ratio of cruising speed to maximum speed? First, express the cruising speed as the numerator of a fraction whose denominator is the maximum speed.

$$\text{Ratio} = \frac{200}{250}$$

Next, reduce the resulting fraction to its simplest form.

$$\text{Ratio} = \frac{200}{250} = \frac{4}{5}$$

Therefore, the ratio of cruising speed to maximum speed is 4:5.

Another common use of ratios is to convert any given ratio to an equivalent ratio with a denominator of 1.

Example: Express the ratio 9:5 as a ratio with a denominator of 1.

$$R = \frac{9}{5} = \frac{?}{1} \quad \text{Since } 9 \div 5 = 1.8, \text{ then } \frac{9}{5} = \frac{1.8}{1}$$

Therefore, 9:5 is the same ratio as 1.8:1. In other words, 9 to 5 is the same ratio as 1.8 to 1.

## Proportion

A proportion is a statement of equality between two or more ratios. For example,

$$\frac{3}{4} = \frac{6}{8} \text{ or } 3:4 = 6:8$$

This proportion is read as, “3 is to 4 as 6 is to 8.”

## Extremes and Means

The first and last terms of the proportion (the 3 and 8 in this example) are called the extremes. The second and third terms (the 4 and 6 in this example) are called the means. In any proportion, the product of the extremes is equal to the product of the means.

In the proportion  $2:3 = 4:6$ , the product of the extremes,  $2 \times 6$ , is 12; the product of the means,  $3 \times 4$ , is also 12. An inspection of any proportion shows this to be true.

## Solving Proportions

Normally when solving a proportion, three quantities are known, and the fourth is unknown. To solve for the unknown, multiply the two numbers along the diagonal and then divide by the third number.

Example: Solve for X in the proportion given below.

$$\frac{65}{80} = \frac{X}{100}$$

First, multiply  $65 \times 100$ :  $65 \times 100 = 6500$

Next, divide by 80:  $6500 \div 80 = 81.25$

Therefore,  $X = 81.25$ .

Example: An airplane flying 300 miles used 24 gallons of gasoline. How many gallons will it need to travel 750 miles?

The ratio here is: “miles to gallons;” therefore, the proportion is set up as:

$$\begin{array}{l} \text{Miles} \\ \text{Gallons} \end{array} \quad \frac{300}{24} = \frac{750}{G}$$

$$\text{Solve for G: } (750 \times 24) \div 300 = 60$$

Therefore, to fly 750 miles, 60 gallons of gasoline is required.

## Percentage

Percentage means “parts out of one hundred.” The percentage sign is “%.” Ninety percent is expressed as 90% (= 90 parts out of 100). The decimal 0.90 equals  $\frac{90}{100}$ , or 90 out of 100, or 90%.

### Expressing a Decimal Number as a Percentage

To express a decimal number in percent, move the decimal point two places to the right (adding zeroes if necessary) and then affix the percent symbol.

Example: Express the following decimal numbers as a percent:

$$0.90 = 90\%$$

$$0.5 = 50\%$$

$$1.25 = 125\%$$

$$0.335 = 33.5\%$$

### Expressing a Percentage as a Decimal Number

Sometimes it may be necessary to express a percentage as a decimal number. To express a percentage as a decimal number, move the decimal point two places to the left and drop the % symbol.

For example: Express the following percentages as decimal numbers:

$$90\% = 0.90$$

$$50\% = 0.50$$

$$5\% = 0.05$$

$$150\% = 1.5$$

### Expressing a Fraction as a Percentage

To express a fraction as a percentage, first change the fraction to a decimal number (by dividing the numerator by the denominator), and then convert the decimal number to a percentage by multiplying by 100 as shown earlier.

Example: Express the fraction  $\frac{5}{8}$  as a percentage.

$$\frac{5}{8} = 5 \div 8 = 0.625 = 62.5\%$$

### Finding a Percentage of a Given Number

This is the most common type of percentage calculation. Here are two methods to solve percentage problems: using algebra or using proportions. Each method is shown next to find a percent of a given number.

Example: In a shipment of 80 wingtip lights, 15% of the lights were defective. How many of the lights were defective?

#### Algebraic Method:

$$15\% \text{ of } 80 \text{ lights} = N \text{ (number of defective lights)}$$

$$0.15 \times 80 = N$$

$$12 = N$$

Therefore, 12 defective lights were in the shipment.

#### Proportion Method:

$$\frac{N}{80} = \frac{15}{100}$$

To solve for N:  $N \times 100 = 80 \times 15$

$$N \times 100 = 1,200$$

$$N = 1,200 \div 100$$

$$N = 12$$

or

$$N = (80 \times 15) \div 100$$

$$N = 12$$

### Finding What Percentage One Number is of Another

Example: A small engine rated at 12 horsepower is found to be delivering only 10.75 horsepower. What is the motor efficiency expressed as a percent?

#### Algebraic Method:

N% of 12 rated horsepower = 10.75 actual horsepower

$$N\% \times 12 = 10.75$$

$$N\% = 10.75 \div 12$$

$$N\% = 0.8958$$

$$N = 89.58$$

Therefore, the motor efficiency is 89.58%.

#### Proportion Method:

$$\frac{10.75}{12} = \frac{N}{100}$$

To solve for N:  $N \times 12 = 10.75 \times 100$

$$N \times 12 = 1,075$$

$$N = 1,075 \div 12$$

$$N = 89.58$$

or

$$N = (1,075 \times 100) \div 12$$

$$N = 89.58$$

Therefore, the motor efficiency is 89.58%.

### Finding a Number When a Percentage of it is Known

Example: Eighty ohms represents 52% of a microphone's total resistance. Find the total resistance of this microphone.

#### Algebraic Method:

$$52\% \text{ of } N = 80 \text{ ohms}$$

$$52\% \times N = 80$$

$$N = 80 \div 0.52$$

$$N = 153.846$$

The total resistance of the microphone is 153.846 ohms.

#### Proportion Method:

$$\frac{80}{N} = \frac{52}{100}$$

To solve for N:  $N \times 52 = 80 \times 100$

$$N \times 52 = 8,000$$

$$N = 8,000 \div 52$$

$$N = 153.846 \text{ ohms}$$

or

$$N = (80 \times 100) \div 52$$

$$N = 153.846 \text{ ohms}$$

## Positive and Negative Numbers (Signed Numbers)

Positive numbers are numbers that are greater than zero. Negative numbers are numbers less than zero. [Figure 3-8] Signed numbers are also called integers.

### Addition of Positive and Negative Numbers

The sum (addition) of two positive numbers is positive. The sum (addition) of two negative numbers is negative. The sum of a positive and a negative number can be positive or negative, depending on the values of the numbers. A good way to visualize a negative number is to think in terms of debt. If you are in debt by \$100 (or,  $-100$ ) and you add \$45 to your account, you are now only \$55 in debt (or  $-55$ ).

Therefore:  $-100 + 45 = -55$ .

Example: The weight of an aircraft is 2,000 pounds. A radio rack weighing 3 pounds and a transceiver weighing 10 pounds are removed from the aircraft. What is the new weight? For weight and balance purposes, all weight removed from an aircraft is given a minus sign, and all weight added is given a plus sign.

$$2,000 + -3 + -10 = 2,000 + -13 = 1,987$$

Therefore, the new weight is 1,987 pounds.

### Subtraction of Positive and Negative Numbers

To subtract positive and negative numbers, first change the “ $-$ ” (subtraction symbol) to a “ $+$ ” (addition symbol), and change the sign of the second number to its opposite (that is, change a positive number to a negative number or vice versa). Finally, add the two numbers together.

Example: The daytime temperature in the city of Denver was  $6^\circ$  below zero ( $-6^\circ$ ). An airplane is cruising at 15,000 feet above Denver. The temperature at 15,000 feet is  $20^\circ$  colder than in the city of Denver. What is the temperature at 15,000 feet?

$$\text{Subtract 20 from } -6: -6 - 20 = -6 + -20 = -26$$

The temperature is  $-26^\circ$ , or  $26^\circ$  below zero at 15,000 feet above the city.

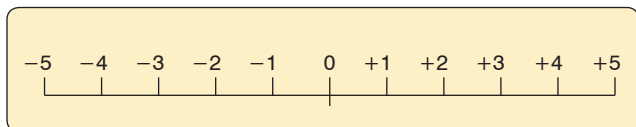


Figure 3-8. A scale of signed numbers.

## Multiplication of Positive and Negative Numbers

The product of two positive numbers is always positive. The product of two negative numbers is always positive. The product of a positive and a negative number is always negative.

Examples:

$$3 \times 6 = 18 \quad -3 \times 6 = -18 \quad -3 \times -6 = 18 \quad 3 \times -6 = -18$$

## Division of Positive and Negative Numbers

The quotient of two positive numbers is always positive. The quotient of two negative numbers is always positive. The quotient of a positive and negative number is always negative.

Examples:

$$6 \div 3 = 2 \quad -6 \div 3 = -2 \quad -6 \div -3 = 2 \quad 6 \div -3 = -2$$

## Powers

The power (or exponent) of a number is a shorthand method of indicating how many times a number, called the base, is multiplied by itself. For example,  $3^4$  is read as “3 to the power of 4.” That is, 3 multiplied by itself 4 times. The 3 is the base and 4 is the power.

Examples:

$$2^3 = 2 \times 2 \times 2 = 8$$

Read “two to the third power equals 8.”

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

Read “ten to the fifth power equals 100,000.”

## Special Powers

### Squared

When a number has a power of 2, it is commonly referred to as “squared.” For example,  $7^2$  is read as “seven squared” or “seven to the second power.” To remember this, think about how a square has two dimensions: length and width.

### Cubed

When a number has a power of 3, it is commonly referred to as “cubed.” For example,  $7^3$  is read as “seven cubed” or “seven to the third power.” To remember this, think about how a cube has three dimensions: length, width, and depth.

### Power of Zero

Any non-zero number raised to the zero power always equals 1.

Example:

$$7^0 = 1 \quad 181^0 = 1 \quad (-24)^0 = 1$$

## Negative Powers

A number with a negative power equals its reciprocal with the same power made positive.

Example: The number  $2^{-3}$  is read as “2 to the negative 3rd power,” and is calculated by:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$$

When using a calculator to raise a negative number to a power, always place parentheses around the negative number (before raising it to a power) so that the entire number gets raised to the power.

## Law of Exponents

When multiplying numbers with powers, the powers can be added as long as the bases are the same.

Example:

$$3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

or  $3^2 \times 3^4 = 3^{(2+4)} = 3^6$

When dividing numbers with powers, the powers can be subtracted as long as the bases are the same.

Example:

$$10^4 \div 10^2 = \frac{10 \times 10 \times 10 \times 10}{10 \times 10} = \frac{\cancel{10} \times \cancel{10} \times 10 \times 10}{\cancel{10} \times \cancel{10}} = 10 \times 10 = 10^2$$

or  $10^4 \div 10^2 = 10^{(4-2)} = 10^2$

## Powers of Ten

Because we use the decimal system of numbers, powers of ten are frequently seen in everyday applications. For example, scientific notation uses powers of ten. Also, many aircraft drawings are scaled to powers of ten. *Figure 3-9* gives more information on the powers of ten and their values.

## Roots

A root is a number that when multiplied by itself a specified number of times produces a given number.

The two most common roots are the square root and the cube root. For more examples of roots, see *Figure 3-10*.

## Square Roots

The square root of 25, written as  $\sqrt{25}$ , equals 5. That is, when the number 5 is squared (multiplied by itself), it produces the number 25. The symbol  $\sqrt{\quad}$  is called a radical sign. Finding the square root of a number is the most common application of roots. The collections of numbers whose square roots are whole numbers are called perfect squares. The first ten perfect squares are: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

Powers of Ten	Expansion	Value
Positive Exponents		
$10^6$	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1,000,000
$10^5$	$10 \times 10 \times 10 \times 10 \times 10$	100,000
$10^4$	$10 \times 10 \times 10 \times 10$	10,000
$10^3$	$10 \times 10 \times 10$	1,000
$10^2$	$10 \times 10$	100
$10^1$	10	10
$10^0$		1
Negative Exponents		
$10^{-1}$	1/10	1/10=0.1
$10^{-2}$	1/(10 x 10)	1/100=0.01
$10^{-3}$	1/(10 x 10 x 10)	1/1,000=0.001
$10^{-4}$	1/(10 x 10 x 10 x 10)	1/10,000=0.0001
$10^{-5}$	1/(10 x 10 x 10 x 10 x 10)	1/100,000=0.00001
$10^{-6}$	1/(10 x 10 x 10 x 10 x 10 x 10)	1/1,000,000=0.000001

**Figure 3-9.** Powers of ten.

The square root of each of these numbers is 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, respectively.

For example,  $\sqrt{36} = 6$  and  $\sqrt{81} = 9$

To find the square root of a number that is not a perfect square, use either a calculator or the estimation method. A longhand method does exist for finding square roots, but with the advent of calculators and because of its lengthy explanation, it is no longer included in this handbook. The estimation method uses the knowledge of perfect squares to approximate the square root of a number.

Example: Find the square root of 31. Since 31 falls between the two perfect roots 25 and 36, we know that must be between  $\sqrt{25}$  and  $\sqrt{36}$ . Therefore,  $\sqrt{31}$  must be greater than 5 and less than 6 because  $\sqrt{25} = 5$  and  $\sqrt{36} = 6$ . If you estimate the square root of 31 at 5.5, you are close to the correct answer. The square root of 31 is actually 5.568.

## Cube Roots

The cube root of 125, written as  $\sqrt[3]{125}$ , equals 5. That is, when the number 5 is cubed (5 multiplied by itself then multiplying the product (25) by 5 again), it produces the number 125. It is common to confuse the “cube” of a number with the “cube root” of a number.

For clarification, the cube of  $27 = 27^3 = 27 \times 27 \times 27 = 19,683$ . However, the cube root of  $27 = \sqrt[3]{27} = 3$ .

Number	Square	Cube	Square Root	Cube Root	Circumference	Area
Number (N)	N Squared (N <sup>2</sup> )	N Cubed (N <sup>3</sup> )	Square Root of N ( $\sqrt{N}$ )	Cube Root of N ( $\sqrt[3]{N}$ )	Circumference of a circle with diameter = N	Area of a circle with diameter = N
1	1	1	1.000	1.000	3.142	0.785
2	4	8	1.414	1.260	6.283	3.142
3	9	27	1.732	1.442	9.425	7.069
4	16	64	2.000	1.587	12.566	12.566
5	25	125	2.236	1.710	15.708	19.635
6	36	216	2.449	1.817	18.850	28.274
7	49	343	2.646	1.913	21.991	38.484
8	64	512	2.828	2.000	25.133	50.265
9	81	729	3.000	2.080	28.274	63.617
10	100	1,000	3.162	2.154	31.416	78.540
11	121	1,331	3.317	2.224	34.558	95.033
12	144	1,728	3.464	2.289	37.699	113.01
13	169	2,197	3.606	2.351	40.841	132.73
14	196	2,744	3.742	2.410	43.982	153.94
15	225	3,375	3.873	2.466	47.124	176.71
16	256	4,096	4.000	2.520	50.265	201.06
17	289	4,913	4.123	2.571	53.407	226.98
18	324	5,832	4.243	2.621	56.549	254.47
19	361	6,859	4.359	2.668	59.690	283.53
20	400	8,000	4.472	2.714	62.832	314.16
21	441	9,261	4.583	2.759	65.973	346.36
22	484	10,648	4.690	2.802	69.115	380.13
23	529	12,167	4.796	2.844	72.257	415.48
24	576	13,824	4.899	2.885	75.398	452.39
25	625	15,625	5.000	2.924	78.540	490.87
26	676	17,576	5.099	2.963	81.681	530.93
27	729	19,683	5.196	3.000	84.823	572.55
28	784	21,952	5.292	3.037	87.965	615.75
29	841	24,389	5.385	3.072	91.106	660.52
30	900	27,000	5.477	3.107	94.248	706.86
31	961	29,791	5.568	3.141	97.389	754.77
32	1,024	32,768	5.657	3.175	100.531	804.25
33	1,089	35,937	5.745	3.208	103.672	855.30
34	1,156	39,304	5.831	3.240	106.814	907.92
35	1,225	42,875	5.916	3.271	109.956	962.11
36	1,296	46,656	6.000	3.302	113.097	1017.88
37	1,369	50,653	6.083	3.332	116.239	1075.21
38	1,444	54,872	6.164	3.362	119.380	1134.11
39	1,521	59,319	6.245	3.391	122.522	1194.59
40	1,600	64,000	6.325	3.420	125.664	1256.64
41	1,681	68,921	6.403	3.448	128.805	1320.25
42	1,764	74,088	6.481	3.476	131.947	1385.44
43	1,849	79,507	6.557	3.503	135.088	1452.20
44	1,936	85,184	6.633	3.530	138.230	1520.53
45	2,025	91,125	6.708	3.557	141.372	1590.43
46	2,116	97,336	6.782	3.583	144.513	1661.90
47	2,209	103,823	6.856	3.609	147.655	1734.94
48	2,304	110,592	6.928	3.634	150.796	1809.56
49	2,401	117,649	7.000	3.659	153.938	1885.74
50	2,500	125,000	7.071	3.684	157.080	1963.49

Figure 3-10. Functions of numbers.

Number	Square	Cube	Square Root	Cube Root	Circumference	Area
Number (N)	N Squared (N <sup>2</sup> )	N Cubed (N <sup>3</sup> )	Square Root of N ( $\sqrt{N}$ )	Cube Root of N ( $\sqrt[3]{N}$ )	Circumference of a circle with diameter = N	Area of a circle with diameter = N
51	2,601	132,651	7.141	3.708	160.221	2042.82
52	2,704	140,608	7.211	3.733	163.363	2123.71
53	2,809	148,877	7.280	3.756	166.504	2206.18
54	2,916	157,464	7.348	3.780	169.646	2290.22
55	3,025	166,375	7.416	3.803	172.787	2375.83
56	3,136	175,616	7.483	3.826	175.929	2463.01
57	3,249	185,193	7.550	3.849	179.071	2551.76
58	3,364	195,112	7.616	3.871	182.212	2642.08
59	3,481	205,379	7.681	3.893	185.354	2733.97
60	3,600	216,000	7.746	3.915	188.495	2827.43
61	3,721	226,981	7.810	3.937	191.637	2922.46
62	3,844	238,328	7.874	3.958	194.779	3109.07
63	3,969	250,047	7.937	3.979	197.920	3117.24
64	4,096	262,144	8.000	4.000	201.062	3216.99
65	4,225	274,625	8.062	4.021	204.203	3318.30
66	4,356	287,496	8.124	4.041	207.345	3421.19
67	4,489	300,763	8.185	4.062	210.487	3525.65
68	4,624	314,432	8.246	4.082	213.628	3631.68
69	4,761	328,509	8.307	4.102	216.770	3739.28
70	4,900	343,000	8.367	4.121	219.911	3848.45
71	5,041	357,911	8.426	4.141	223.053	3959.19
72	5,184	373,248	8.485	4.160	226.194	4071.50
73	5,329	389,017	8.544	4.179	229.336	4185.38
74	5,476	405,224	8.602	4.198	232.478	4300.84
75	5,625	421,875	8.660	4.217	235.619	4417.86
76	5,776	438,976	8.718	4.236	238.761	4536.46
77	5,929	456,533	8.775	4.254	241.902	4656.62
78	6,084	474,552	8.832	4.273	245.044	4778.36
79	6,241	493,039	8.888	4.291	248.186	4901.67
80	6,400	512,000	8.944	4.309	251.327	5026.54
81	6,561	531,441	9.000	4.327	254.469	5152.99
82	6,724	551,368	9.055	4.344	257.610	5281.01
83	6,889	571,787	9.110	4.362	260.752	5410.60
84	7,056	592,704	9.165	4.380	263.894	5541.76
85	7,225	614,125	9.220	4.397	267.035	5674.50
86	7,396	636,056	9.274	4.414	270.177	5808.80
87	7,569	658,503	9.327	4.431	273.318	5944.67
88	7,744	681,472	9.381	4.448	276.460	6082.12
89	7,921	704,969	9.434	4.465	279.602	6221.13
90	8,100	729,000	9.487	4.481	282.743	6361.72
91	8,281	753,571	9.539	4.498	285.885	6503.88
92	8,464	778,688	9.592	4.514	289.026	6647.60
93	8,649	804,357	9.644	4.531	292.168	6792.90
94	8,836	830,584	9.695	4.547	295.309	6939.77
95	9,025	857,375	9.747	4.563	298.451	7088.21
96	9,216	884,736	9.798	4.579	301.593	7238.22
97	9,409	912,673	9.849	4.595	304.734	7389.81
98	9,604	941,192	9.900	4.610	307.876	7542.96
99	9,801	970,299	9.950	4.626	311.017	7697.68
100	10,000	1,000,000	10.000	4.642	314.159	7853.98

Figure 3-10. Functions of numbers (continued).

## Fractional Powers

Another way to write a root is to use a fraction as the power (or exponent) instead of the radical sign. The square root of a number is written with a  $\frac{1}{2}$  as the exponent instead of a radical sign. The cube root of a number is written with an exponent of  $\frac{1}{3}$  and the fourth root with an exponent of  $\frac{1}{4}$  and so on.

$$\text{Example: } \sqrt{31} = 31^{\frac{1}{2}} \quad \sqrt[3]{125} = 125^{\frac{1}{3}} \quad \sqrt[4]{16} = 16^{\frac{1}{4}}$$

## Functions of Numbers Chart

The Functions of Numbers chart found in *Figure 3-10* is included in this chapter for convenience in making computations. Each column in the chart is listed below, with new concepts explained.

- Number (N)
- N squared ( $N^2$ )
- N cubed ( $N^3$ )
- Square root of N ( $\sqrt{N}$ )
- Cube root of N ( $\sqrt[3]{N}$ )
- Circumference of a circle with diameter = N. Circumference is the linear measurement of the distance around a circle. The circumference is calculated by multiplying the diameter of the circle by 3.1416 (3.1416 is the number referred to as pi, which has the symbol  $\pi$ ). If the diameter of a circle is 10 inches, then the circumference would be:  
 $10 \times 3.1416 = 31.4160$ .
- Area of a circle with diameter = N. Area of a circle is the number of square units of measurement contained in the circle with a diameter of N. The area of a circle equals  $\pi$  multiplied by the radius squared. This is calculated by the formula:  $A = \pi \times r^2$ . Remember that the radius is equal to one-half of the diameter.

Example: A flight deck instrument gauge has a round face that is 3 inches in diameter. What is the area of the face of the gauge? From *Figure 3-10* for N = 3, the answer is 7.0686 square inches. This is calculated by:

If the diameter of the gauge is 3 inches, then the radius =  $\frac{1}{2} \times 3 = 1.5$  inches.

$$\text{Area} = \pi \times r^2 = 3.1416 \times 1.5^2 = 3.1416 \times 2.25 = 7.0686 \text{ square inches.}$$

## Scientific Notation

Scientific notation is used as a type of shorthand to express very large or very small numbers. It is a way to write numbers so that they do not take up as much space on the page. The format of a number written in scientific notation has two parts. The first part is a number greater than or equal to 1 and less than 10 (for example, 2.35). The second part is a power of 10 (for

example,  $10^6$ ). The number 2,350,000 is expressed in scientific notation as  $2.35 \times 10^6$ . It is important that the decimal point is always placed to the right of the first digit. Notice that very large numbers always have a positive power of 10 and very small numbers always have a negative power of 10.

Example: The velocity of the speed of light is over 186,000 miles per second (mps). This can be expressed as  $1.86 \times 10^5$  mps in scientific notation. The mass of an electron is approximately 0.000,000,000,000,000,000,000,000,911 grams. This can be expressed in scientific notation as  $9.11 \times 10^{-28}$  grams.

## Converting Numbers from Standard Notation to Scientific Notation

Example: Convert 1,244,000,000,000 to scientific notation as follows. First, note that the decimal point is to the right of the last zero. (Even though it is not usually written, it is assumed to be there.)

$$1,244,000,000,000 = 1,244,000,000,000.0$$

To change to the format of scientific notation, the decimal point must be moved to the position between the first and second digits. In this case, it is between the 1 and the 2. Since the decimal point must be moved 12 places to the left to get there, the power of 10 is 12. Remember that large numbers always have a positive exponent. Therefore,  $1,244,000,000,000 = 1.244 \times 10^{12}$  when written in scientific notation.

Example: Convert 0.000000457 from standard notation to scientific notation. To change to the format of scientific notation, the decimal point must be moved to the position between the first and second numbers, which in this case is between the 4 and the 5. Since the decimal point must be moved 7 places to the right to get there, the power of 10 is  $-7$ . Remember that small numbers (those less than one) have a negative exponent. Therefore,  $0.000000457 = 4.57 \times 10^{-7}$  when written in scientific notation.

## Converting Numbers from Scientific Notation to Standard Notation

Example: Convert  $3.68 \times 10^7$  from scientific notation to standard notation, as follows. To convert from scientific notation to standard notation, move the decimal place 7 places to the right.  $3.68 \times 10^7 = 36,800,000$ . Another way to think about the conversion is  $3.68 \times 10^7 = 3.68 \times 10,000,000 = 36,800,000$ .

Example: Convert  $7.1543 \times 10^{-10}$  from scientific notation to standard notation. Move the decimal place 10 places to the left:  $7.1543 \times 10^{-10} = 0.00000000071543$ . Another way to think about the conversion is  $7.1543 \times 10^{-10} = 7.1543 \times 0.0000000001 = 0.00000000071543$



When converting, remember that large numbers always have positive powers of ten and small numbers always have negative powers of ten. Refer to *Figure 3-11* to determine which direction to move the decimal point.

### Addition, Subtraction, Multiplication, and Division of Scientific Numbers

To add, subtract, multiply, or divide numbers in scientific notation, change the scientific notation number back to standard notation. Then add, subtract, multiply or divide the standard notation numbers. After the computation, change the final standard notation number back to scientific notation.

### Algebra

Algebra is the branch of mathematics that uses letters or symbols to represent variables in formulas and equations.

For example, in the equation  $d = v \times t$ , where distance = velocity  $\times$  time, the variables are: d, v, and t.

### Equations

Algebraic equations are frequently used in aviation to show the relationship between two or more variables. Equations normally have an equals sign (=) in the expression.

Example: The formula  $A = \pi \times r^2$  shows the relationship between the area of a circle (A) and the length of the radius (r) of the circle. The area of a circle is equal to  $\pi$  (3.1416) times the radius squared. The larger the radius, the larger the area of the circle.

### Algebraic Rules

When solving for a variable in an equation, you can add, subtract, multiply, or divide the terms in the equation (you do the same to both sides of the equals sign) to get the variable onto one side of the equals sign.

Examples: Solve the following equations for the value N.

$$3N = 21$$

Conversion	Large Numbers with Positive Powers of 10	Small Numbers with Negative Powers of 10
From standard notation to scientific notation	Move decimal place to the left	Move decimal place to the right
From scientific notation to standard notation	Move decimal place to the right	Move decimal place to the left

Figure 3-11. Converting between scientific and standard notation.

To solve for N, divide both sides by 3.

$$3N \div 3 = 21 \div 3$$

$$N = 7$$

$$N + 17 = 59$$

To solve for N, subtract 17 from both sides.

$$N + 17 - 17 = 59 - 17$$

$$N = 42$$

$$N - 22 = 100$$

To solve for N, add 22 to both sides.

$$N - 22 + 22 = 100 + 22$$

$$N = 122$$

$$\frac{N}{5} = 50$$

To solve for N, multiply both sides by 5.

$$N \times 5 = 50 \times 5$$

$$N = 250$$

### Solving for a Variable

Another application of algebra is to solve an equation for a given variable.

Example: Using the formula given in *Figure 3-12*, find the total capacitance ( $C_T$ ) of the series circuit containing three capacitors with

$$C_1 = 0.1 \text{ microfarad}$$

$$C_2 = 0.015 \text{ microfarad}$$

$$C_3 = 0.05 \text{ microfarad}$$

First, substitute the given values into the formula:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{0.1} + \frac{1}{0.015} + \frac{1}{0.05}} = \frac{1}{10 + 66.66 + 20}$$

Therefore,  $C_T = 1/96.66 = 0.01034$  microfarad. The microfarad ( $10^{-6}$  farad) is a unit of measurement of capacitance. This is discussed in greater length in Chapter 12, Electricity.

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Figure 3-12. Total capacitance in a series circuit.

## Use of Parentheses

In algebraic equations, parentheses are used to group numbers or symbols together. The use of parentheses helps us to identify the order in which we should apply mathematical operations. The operations inside the parentheses are always performed first in algebraic equations.

Example: Solve the algebraic equation  $X = (4 + 3)^2$ . First, perform the operation inside the parentheses, which is,  $4 + 3 = 7$ . Then complete the exponent calculation  $X = (7)^2 = 7 \times 7 = 49$ .

When using more complex equations, which may combine several terms and use multiple operations, grouping the terms together helps organize the equation. Parentheses, ( ), are most commonly used in grouping, but you may also see brackets, [ ]. When a term or expression is inside one of these grouping symbols, it means that any operation indicated to be done on the group is done to the entire term or expression.

Example:

Solve the equation  $N = 2 \times [(9 \div 3) + (4 + 3)^2]$ . Start with the operations inside the parentheses ( ), then perform the operations inside the brackets [ ].

$$N = 2 \times [(9 \div 3) + (4 + 3)^2]$$
$$N = 2 \times [3 + (7)^2]$$

First, complete the operations inside the parentheses ( ).

$$N = 2 \times [3 + 49]$$
$$N = 2 \times [52]$$

Second, complete the operations inside the brackets [ ].

$$N = 104$$

## Order of Operation

In algebra, rules have been set for the order in which operations are evaluated. These same universally accepted rules are also used when programming algebraic equations in calculators. When solving the following equation, the order of operation is given below:

$$N = (62 - 54)^2 + 6^2 - 4 + 3 \times [8 + (10 \div 2)] + \sqrt{25} + (42 \times 2) \div 4 + \frac{3}{4}$$

1. **Parentheses.** First you must do everything in parentheses, ( ), starting from the innermost parentheses. If the expression has a set of brackets, [ ], treat these exactly like parentheses. If you are working with a fraction, treat the top as if it was in parentheses and the denominator as if it were in parentheses, even if there is none shown. From the equation above, completing the calculation in parentheses gives the following:

$$N = (8)^2 + 6^2 - 4 + 3 \times [8 + (5)] + \sqrt{25} + (84) \div 4 + \frac{3}{4}$$

then

$$N = (8)^2 + 6^2 - 4 + 3 \times [13] + \sqrt{25} + 84 \div 4 + \frac{3}{4}$$

2. **Exponents.** Next, clear any exponents. Treat any roots (square roots, cube roots, and so forth) as exponents. Completing the exponents and roots in the equation gives the following:

$$N = 64 + 36 - 4 + 3 \times 13 + 5 + 84 \div 4 + \frac{3}{4}$$

3. **Multiplication and Division.** Evaluate all of the multiplications and divisions from left to right. Multiply and divide from left to right in one step. A common error is to use two steps for this (that is, to clear all of the multiplication signs and then clear all of the division signs), but that is not the correct method. Treat fractions as division. Completing the multiplication and division in the equation gives the following:

$$N = 64 + 36 - 4 + 39 + 5 + 21 + \frac{3}{4}$$

4. **Addition and Subtraction.** Evaluate the additions and subtractions from left to right. Like above, addition and subtraction are computed left to right in one step. Completing the addition and subtraction in the equation gives the following:

$$X = 161 \frac{3}{4}$$

A commonly used acronym, PEMDAS, is used for remembering the order of operation in algebra. PEMDAS is an acronym for parentheses, exponents, multiplication, division, addition, and subtraction. To remember it, many use the sentence, "Please Excuse My Dear Aunt Sally." Always remember, however, to multiply/divide or add/subtract in one sweep from left to right, not separately.

## Order of Operation for Algebraic Equations

1. **Parentheses**
2. **Exponents**
3. **Multiplication**
4. **Division**
5. **Addition**
6. **Subtraction**

## Computing Area of Two-Dimensional Solids

Area is a measurement of the amount of surface of an object. Area is usually expressed in such units as square inches or square centimeters for small surfaces or in square feet or square meters for larger surfaces.

*Figure 3-13* summarizes the formulas for computing the area of two-dimensional solids.

Object	Area	Formula	Figure
Rectangle	length × width	$a = l \times w$	3-14
Square	length × width or side × side	$a = l \times w$ or $a = s^2$	3-15
Triangle	$\frac{1}{2} \times (\text{length} \times \text{height})$ or $\frac{1}{2} \times (\text{base} \times \text{height})$ or $(\text{base} \times \text{height}) \div 2$	$a = \frac{1}{2} (l \times h)$ or $a = \frac{1}{2} (b \times h)$ or $a = (b \times h) \div 2$	3-16 3-17
Parallelogram	length × height	$a = l \times h$	3-18
Trapezoid	$\frac{1}{2} (\text{base}_1 + \text{base}_2) \times \text{height}$	$a = \frac{1}{2} (b_1 + b_2) \times h$	3-19
Circle	$\pi \times \text{radius}^2$	$a = \pi \times r^2$	3-20
Ellipse	$\pi \times \text{semi-axis A} \times \text{semi-axis B}$	$a = \pi \times a \times b$	3-21
Wing area	span × mean chord	$a = s \times c$	3-22

Figure 3-13. Formulas to compute area.

### Rectangle

A rectangle is a four-sided figure with opposite sides of equal length and parallel to each other. [Figure 3-14] All of the angles are right angles. A right angle is a 90° angle. The rectangle is a very familiar shape in mechanics. The formula for the area of a rectangle is:

$$\text{area} = \text{length} \times \text{width} = l \times w$$

Example: An aircraft floor panel is in the form of a rectangle having a length of 24 inches and a width of 12 inches. What is the area of the panel expressed in square inches? First, determine the known values and substitute them in the formula.

$$a = l \times w = 24 \text{ inches} \times 12 \text{ inches} = 288 \text{ square inches}$$

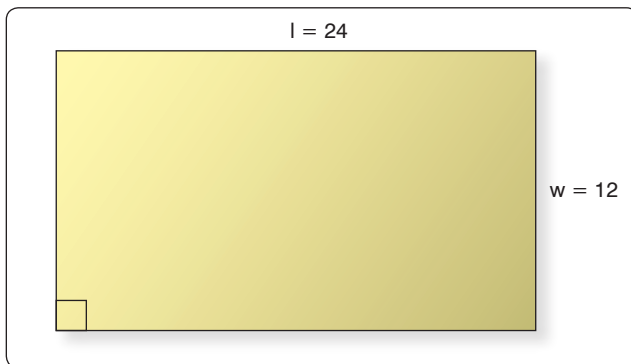


Figure 3-14. Rectangle.

### Square

A square is a four-sided figure with all sides of equal length and opposite sides are parallel to each other. [Figure 3-15] All angles are right angles. A right angle is a 90° angle. The formula for the area of a square is:

$$\text{area} = \text{length} \times \text{width} = l \times w$$

Since the length and the width of a square are the same value, the formula for the area of a square can also be written as:

$$\text{area} = \text{side} \times \text{side} = s^2$$

Example: What is the area of a square access plate whose side measures 25 inches? First, determine the known value and substitute it in the formula.

$$a = l \times w = 25 \text{ inches} \times 25 \text{ inches} = 625 \text{ square inches}$$

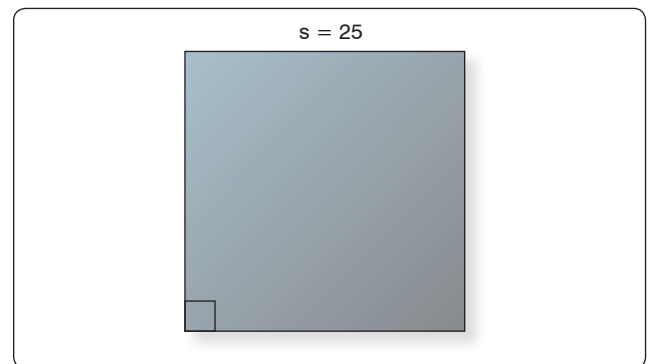


Figure 3-15. Square.

### Triangle

A triangle is a three-sided figure. The sum of the three angles in a triangle is always equal to 180°. Triangles are often classified by their sides. An equilateral triangle has 3 sides of equal length. An isosceles triangle has 2 sides of equal length. A scalene triangle has three sides of differing lengths.

Triangles can also be classified by their angles. An acute triangle has all three angles less than 90°. A right triangle has one right angle (a 90° angle). An obtuse triangle has







Triangles Based on Sides		
<b>Scalene</b>  Length of all sides are different	<b>Isosceles</b>  Length of two sides are equal	<b>Equilateral</b>  Length of all sides are equal
Triangles Based on Angles		
<b>Acute</b>  Each angle is $< 90^\circ$	<b>Right</b>  One angle is $= 90^\circ$	<b>Obtuse</b>  One angle is $> 90^\circ$

Figure 3-16. Types of triangles.

one angle greater than  $90^\circ$ . Each of these types of triangles is shown in Figure 3-16.

The formula for the area of a triangle is

$$\text{area} = \frac{1}{2} \times (\text{base} \times \text{height}) = \frac{1}{2} \times (b \times h)$$

Example: Find the area of the obtuse triangle shown in Figure 3-17. First, substitute the known values in the area formula.

$$a = \frac{1}{2} \times (b \times h) = \frac{1}{2} \times (2'6" \times 3'2")$$

Next, convert all dimensions to inches:

$$2'6" = (2 \times 12") + 6" = (24 + 6) = 30 \text{ inches}$$

$$3'2" = (3 \times 12") + 2" = (36 + 2) = 38 \text{ inches}$$

Now, solve the formula for the unknown value:

$$a = \frac{1}{2} \times (30 \text{ inches} \times 38 \text{ inches}) = 570 \text{ square inches}$$

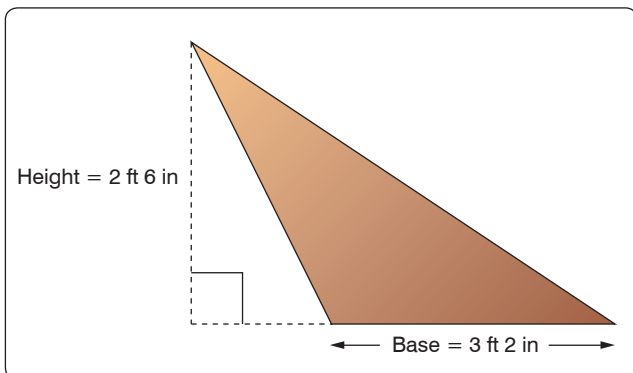


Figure 3-17. Obtuse triangle.

## Parallelogram

A parallelogram is a four-sided figure with two pairs of parallel sides. [Figure 3-18] Parallelograms do not necessarily have four right angles.

The formula for the area of a parallelogram is:

$$\text{area} = \text{length} \times \text{height} = l \times h$$

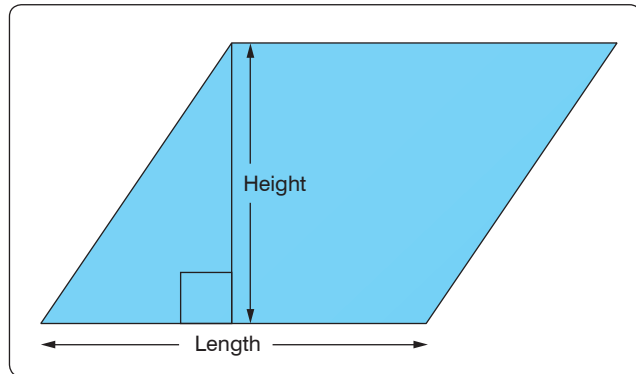


Figure 3-18. Parallelogram.

## Trapezoid

A trapezoid is a four-sided figure with one pair of parallel sides. [Figure 3-19] The formula for the area of a trapezoid is:

$$\text{area} = \frac{1}{2} (\text{base}_1 + \text{base}_2) \times \text{height}$$

Example: What is the area of a trapezoid in Figure 3-19 whose bases are 14 inches and 10 inches, and whose height (or altitude) is 6 inches? First, substitute the known values in the formula.

$$\begin{aligned} a &= \frac{1}{2} (b_1 + b_2) \times h \\ &= \frac{1}{2} (14 \text{ inches} + 10 \text{ inches}) \times 6 \text{ inches} \end{aligned}$$

$$\begin{aligned} a &= \frac{1}{2} (24 \text{ inches}) \times 6 \text{ inches} \\ &= 12 \text{ inches} \times 6 \text{ inches} = 72 \text{ square inches} \end{aligned}$$

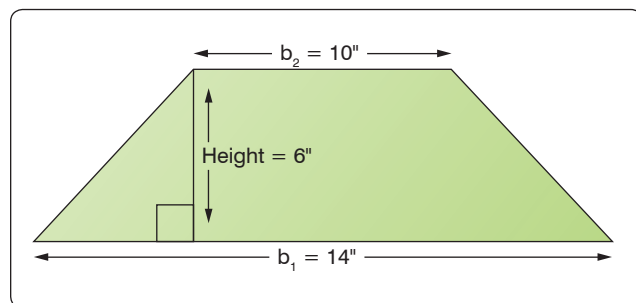


Figure 3-19. Trapezoid.

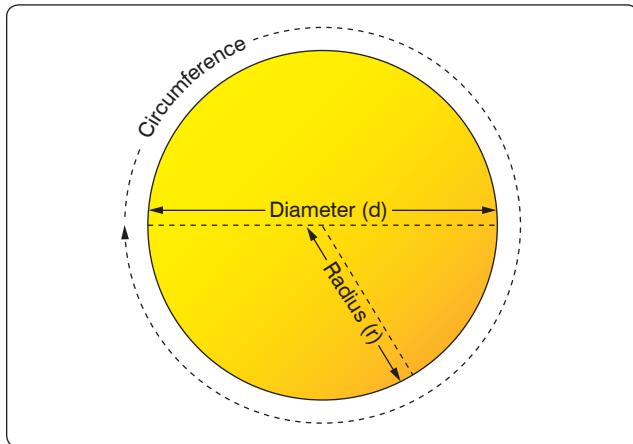


Figure 3-20. Circle.

### Circle

A circle is a closed, curved, plane figure. [Figure 3-20] Every point on the circle is an equal distance from the center of the circle. The diameter is the distance across the circle (through the center). The radius is the distance from the center to the edge of the circle. The diameter is always twice the length of the radius. The circumference, or distance around, a circle is equal to the diameter times  $\pi$ .

$$\text{circumference} = c = d \pi$$

The formula for the area of a circle is:

$$\text{area} = \pi \times \text{radius}^2 = \pi \times r^2$$

Example: The bore, which is “inside diameter,” of a certain aircraft engine cylinder is 5 inches. Find the area of the cross section of the cylinder.

First, substitute the known values in the formula:

$$a = \pi \times r^2$$

The diameter is 5 inches, so the radius is 2.5 inches. (diameter = radius  $\times$  2)

$$a = 3.1416 \times (2.5 \text{ inches})^2 = 3.1416 \times 6.25 \text{ square inches} = 19.635 \text{ square inches}$$

### Ellipse

An ellipse is a closed, curved, plane figure and is commonly called an oval. [Figure 3-21] In a radial engine, the articulating rods connect to the hub by pins, which travel in the pattern of an ellipse (i.e., an elliptical or orbital path).

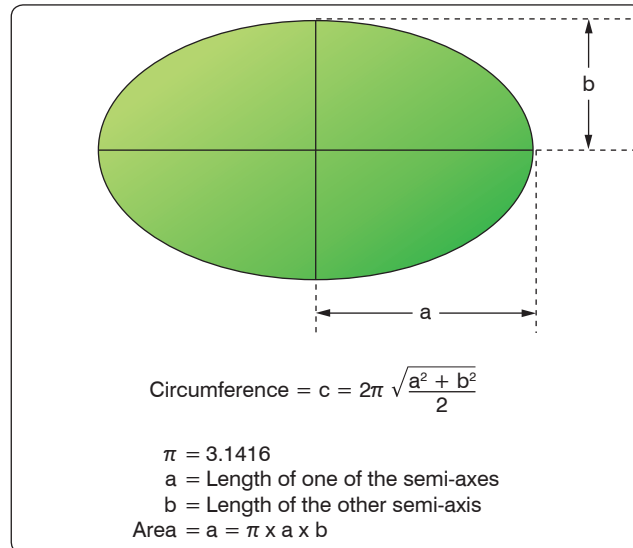


Figure 3-21. Ellipse.

### Wing Area

To describe the shape of a wing [Figure 3-22], several terms are required. To calculate wing area, it is necessary to know the meaning of the terms “span” and “chord.” The wingspan, S, is the length of the wing from wingtip to wingtip. The chord is the average width of the wing from leading edge to trailing edge. If the wing is a tapered wing, the average width, known as the mean chord (C), must be known to find the area. The formula for calculating wing area is:

$$\text{area of a wing} = \text{span} \times \text{mean chord}$$

Example: Find the area of a tapered wing whose span is 50 feet and whose mean chord is 6'8". First, substitute the known values in the formula.

$$\begin{aligned} a &= s \times c \\ &= 50 \text{ feet} \times 6 \text{ feet } 8 \text{ inches} \\ &\quad (\text{Note: } 8 \text{ inches} = \frac{8}{12} \text{ feet} = 0.67 \text{ feet}) \\ &= 50 \text{ feet} \times 6.67 \text{ feet} \\ &= 333.5 \text{ square feet} \end{aligned}$$

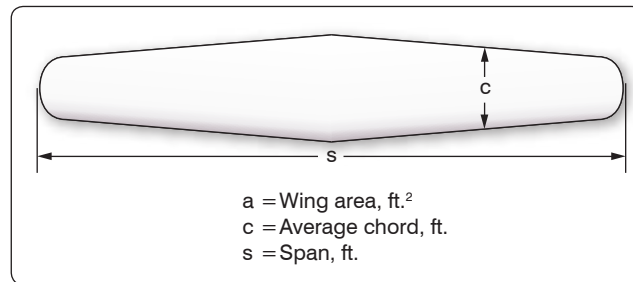


Figure 3-22. Wing planform.

## Units of Area

A square foot measures 1 foot by 1 foot. It also measures 12 inches by 12 inches. Therefore, one square foot also equals 144 square inches (that is,  $12 \times 12 = 144$ ). To convert square feet to square inches, multiply by 144. To convert square inches to square feet, divide by 144.

A square yard measures 1 yard by 1 yard. It also measures 3 feet by 3 feet. Therefore, one square yard also equals 9 square feet (that is,  $3 \times 3 = 9$ ). To convert square yards to square feet, multiply by 9. To convert square feet to square yards, divide by 9. Refer to *Figure 3-23*, Applied Mathematics Formula Sheet, for a comparison of different units of area.

## Computing Volume of Three-Dimensional Solids

Three-dimensional solids have length, width, and height. There are many three-dimensional solids, but the most common are rectangular solids, cubes, cylinders, spheres, and cones. Volume is the amount of space within a solid. Volume is expressed in cubic units. Cubic inches or cubic centimeters are used for small spaces and cubic feet or cubic meters for larger spaces.

### Conversion Factors

Length			Area		
1 inch	2.54 centimeters	25.4 millimeters	1 square inch	6.45 square centimeters	
1 foot	12 inches	30.48 centimeters	1 square foot	144 square inches	0.093 square meters
1 yard	3 feet	0.9144 meters	1 square yard	9 square feet	0.836 square meters
1 mile	5,280 feet	1,760 yards	1 acre	43,560 square feet	
1 millimeter	0.0394 inches		1 square mile	640 acres	2.59 square kilometers
1 kilometer	0.62 miles		1 square centimeter	0.155 square inches	
			1 square meter	1.195 square yards	
			1 square kilometer	0.384 square miles	

Volume					
1 fluid ounce	29.57 cubic centimeters				
1 cup	8 fluid ounces				
1 pint	2 cups	16 fluid ounces	0.473 liters		
1 quart	2 pints	4 cups	32 fluid ounces	0.9463 liters	
1 gallon	4 quarts	8 pints	16 cups	128 ounces	3.785 liters
1 gallon	231 cubic inches				
1 liter	0.264 gallons	1.057 quarts			
1 cubic foot	1,728 cubic inches	7.5 gallons			
1 cubic yard	27 cubic feet				
1 board foot	1 inch x 12 inches x 12 inches				

Weight				Temperature	
1 ounce	28.350 grams			°F to °C	Celsius = $\frac{5}{9} \times (°F) - 32$
1 pound	16 ounces	453.592 grams	0.4536 kilograms	°C to °F	Fahrenheit = $\frac{9}{5} \times (°C) + 32$
1 ton	2,000 pounds				
1 milligram	0.001 grams				
1 kilogram	1,000 grams	2.2 pounds			
1 gram	0.0353 ounces				

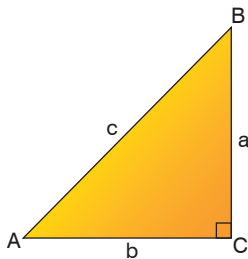
Figure 3-23. Applied mathematics formula sheet.

## Order of Operation for Algebraic Equations

1. Parentheses
2. Exponents
3. Multiplication
4. Division
5. Addition
6. Subtraction

Use the acronym PEMDAS to remember the order of operation in algebra. PEMDAS is an acronym for parentheses, exponents, multiplication, division, addition, and subtraction. To remember it, many use the sentence, "Please Excuse My Dear Aunt Sally."

## Trigonometric Equations

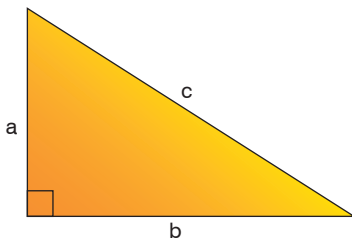


$$\text{Sine (sin) of angle A} = \frac{\text{opposite side (side a)}}{\text{hypotenuse (side c)}}$$

$$\text{Cosine (cos) of angle A} = \frac{\text{adjacent side (side b)}}{\text{hypotenuse (side c)}}$$

$$\text{Tangent (tan) of angle A} = \frac{\text{opposite side (side a)}}{\text{adjacent side (side b)}}$$

## Pythagorean Theorem



$$a^2 + b^2 = c^2$$

Figure 3-23. Applied mathematics formula sheet (continued).

### Rectangular Solid

A rectangular solid is a three-dimensional solid with six rectangular-shaped sides. [Figure 3-24] The volume is the number of cubic units within the rectangular solid. The formula for the volume of a rectangular solid is:

$$\text{volume} = \text{length} \times \text{width} \times \text{height} = l \times w \times h$$

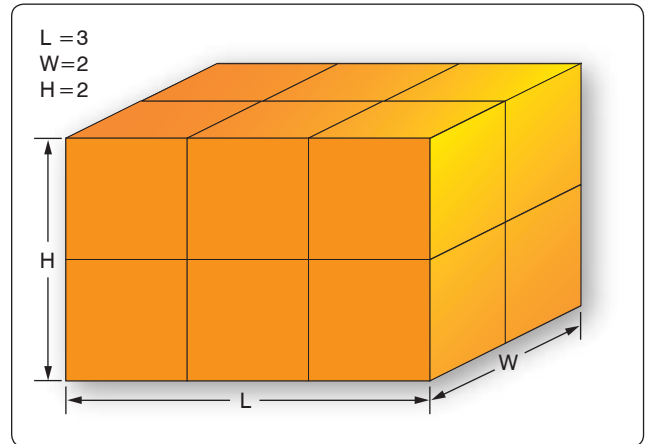


Figure 3-24. Rectangular solid.

In Figure 3-25, the rectangular solid is 3 feet by 2 feet by 2 feet.

The volume of the solid in Figure 3-24 is  $= 3 \text{ ft} \times 2 \text{ ft} \times 2 \text{ ft} = 12 \text{ cubic feet}$ .

Example: A rectangular baggage compartment measures 5 feet 6 inches in length, 3 feet 4 inches in width, and 2 feet 3 inches in height. How many cubic feet of baggage will it hold? First, substitute the known values into the formula.

$$\begin{aligned} v &= l \times w \times h \\ &= 5'6" \times 3'4" \times 2'3" \\ &= 5.5 \text{ ft} \times 3.33 \text{ ft} \times 2.25 \text{ ft} \\ &= 41.25 \text{ cubic feet} \end{aligned}$$

### Cube

A cube is a solid with six square sides. [Figure 3-25] A cube is just a special type of rectangular solid. It has the same formula for volume as does the rectangular solid, which is  $\text{volume} = \text{length} \times \text{width} \times \text{height} = L \times W \times H$ . Because all of the sides of a cube are equal, the volume formula for a cube can also be written as:

$$\text{volume} = \text{side} \times \text{side} \times \text{side} = S^3$$

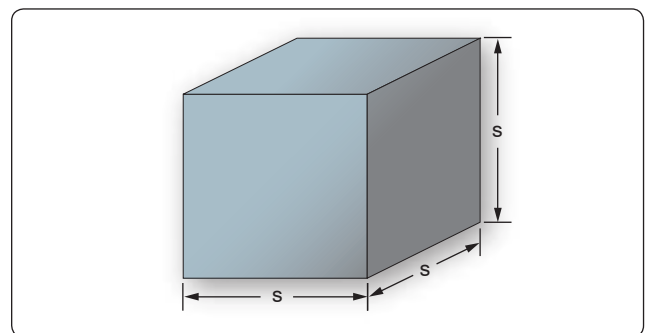


Figure 3-25. Cube.

Example: A large, cube-shaped carton contains a shipment of smaller boxes inside of it. Each of the smaller boxes is 1 ft × 1 ft × 1 ft. The measurement of the large carton is 3 ft × 3 ft × 3 ft. How many of the smaller boxes are in the large carton? First, substitute the known values into the formula.

$$\begin{aligned}
 v &= l \times w \times h \\
 &= 3 \text{ ft} \times 3 \text{ ft} \times 3 \text{ ft} \\
 &= 27 \text{ cubic feet of volume in the} \\
 &\quad \text{large carton}
 \end{aligned}$$

Since each of the smaller boxes has a volume of 1 cubic foot, the large carton holds 27 boxes.

### Cylinder

A solid having the shape of a can, a length of pipe, or a barrel is called a cylinder. [Figure 3-26] The ends of a cylinder are identical circles. The formula for the volume of a cylinder is:

$$\text{volume} = \pi \times \text{radius}^2 \times \text{height of the cylinder} = \pi r^2 \times h$$

One of the most important applications of the volume of a cylinder is finding the piston displacement of a cylinder in a reciprocating engine. Piston displacement is the total volume (in cubic inches, cubic centimeters, or liters) swept by all of the pistons of a reciprocating engine as they move in one revolution of the crankshaft. The formula for piston displacement is given as:

$$\begin{aligned}
 \text{Piston Displacement} &= \\
 \pi \times (\text{bore divided by } 2)^2 \times \text{stroke} \times (\# \text{ cylinders})
 \end{aligned}$$

The bore of an engine is the inside diameter of the cylinder. The stroke of the engine is the length the piston travels inside the cylinder. [Figure 3-27]

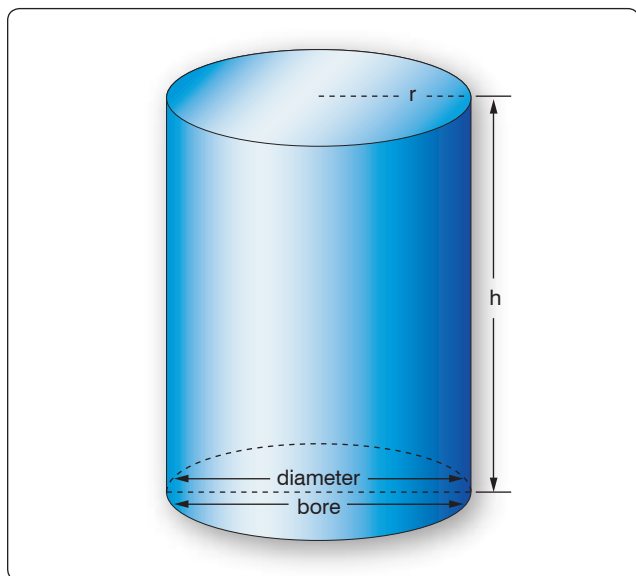


Figure 3-26. Cylinder.

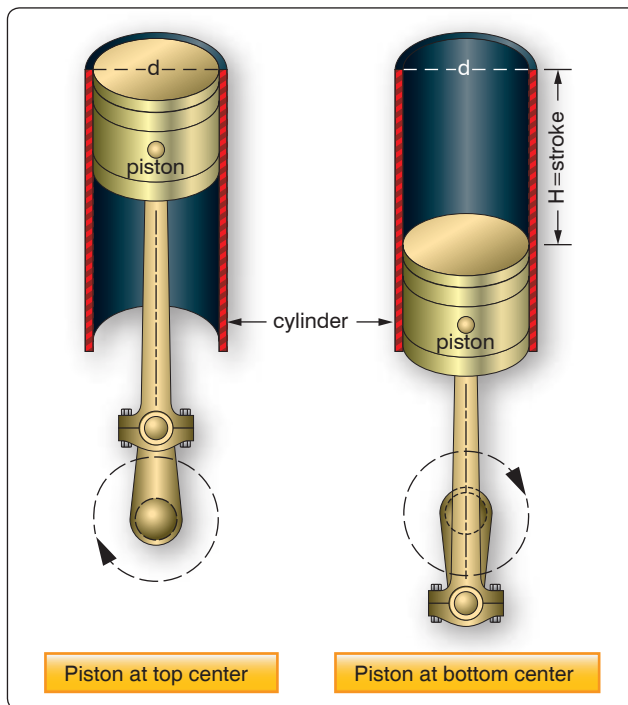


Figure 3-27. Cylinder displacement.

Example: Find the piston displacement of one cylinder in a multi-cylinder aircraft engine. The engine has a cylinder bore of 5.5 inches and a stroke of 5.4 inches. First, substitute the known values in the formula.

$$v = \pi \times r^2 \times h = (3.1416) \times (5.5 \div 2)^2 \times (5.4)$$

$$v = 23.758 \times 5.4 = 128.29 \text{ cubic inches}$$

The piston displacement of one cylinder is 128.29 cubic inches. For an eight-cylinder engine, then the total engine displacement would be:

$$\begin{aligned}
 \text{Total displacement for 8 cylinders} &= 8 \times 128.29 = \\
 &1026.32 \text{ cubic inches of displacement}
 \end{aligned}$$

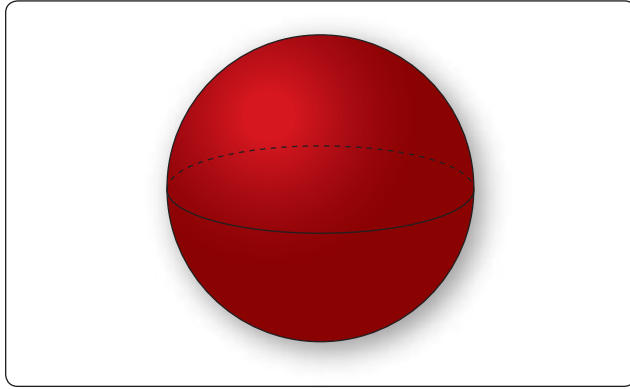
### Sphere

A solid having the shape of a ball is called a sphere. [Figure 3-28] A sphere has a constant diameter. The radius (r) of a sphere is one-half of the diameter (d). The formula for the volume of a sphere is given as:

$$v = \frac{4}{3} \times \pi \times \text{radius}^3 = \frac{4}{3} \times \pi \times r^3 \quad \text{or} \quad v = \frac{1}{6} \times \pi d^3$$

Example: A pressure tank inside the fuselage of a cargo aircraft is in the shape of a sphere with a diameter of 34 inches. What is the volume of the pressure tank?





**Figure 3-28.** Sphere.

$$\begin{aligned}
 v &= \frac{4}{3} \times \pi \times \text{radius}^3 = \frac{4}{3} \times (3.1416) \times (3\frac{1}{2})^3 \\
 &= 1.33 \times 3.1416 \times 17^3 = 1.33 \times 3.1416 \times 4,913 \\
 v &= 20,528.125 \text{ cubic inches}
 \end{aligned}$$

### Cone

A solid with a circle as a base and with sides that gradually taper to a point is called a cone. [Figure 3-29] The formula for the volume of a cone is given as:

$$v = \frac{1}{3} \times \pi \times \text{radius}^2 \times \text{height} = \frac{1}{3} \times \pi \times r^2 \times h$$

### Units of Volume

Since all volumes are not measured in the same units, it is necessary to know all the common units of volume and how they are related to each other. For example, the mechanic may know the volume of a tank in cubic feet or cubic inches, but when the tank is full of gasoline, he or she is interested in how many gallons it contains. Refer to *Figure 3-23*, Applied Mathematics Formula Sheet, for a comparison of different units of volume.

## Computing Surface Area of Three-Dimensional Solids

The surface area of a three-dimensional solid is the sum of the areas of the faces of the solid. Surface area is a different concept from that of volume. For example, surface area is the amount of sheet metal needed to build a rectangular fuel tank while volume is the amount of fuel that the tank can contain.

### Rectangular Solid

The formula for the surface area of a rectangular solid [Figure 3-24] is given as:

$$\begin{aligned}
 \text{Surface area} &= \\
 &2 \times [(\text{width} \times \text{length}) + (\text{width} \times \text{height}) + (\text{length} \times \text{height})] \\
 &= 2 \times [(w \times l) + (w \times h) + (l \times h)]
 \end{aligned}$$

### Cube

The formula for the surface area of a cube [Figure 3-25] is given as:

$$\text{Surface area} = 6 \times (\text{side} \times \text{side}) = 6 \times s^2$$

Example: What is the surface area of a cube with a side measure of 8 inches?

$$\begin{aligned}
 \text{Surface area} &= 6 \times (\text{side} \times \text{side}) \\
 &= 6 \times S^2 = 6 \times 8^2 = 6 \times 64 \\
 &= 384 \text{ square inches}
 \end{aligned}$$

### Cylinder

The formula for the surface area of a cylinder [Figure 3-26] is given as:

$$\begin{aligned}
 \text{Surface area} &= 2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height} \\
 &= 2 \times \pi \times r^2 + \pi \times d \times h
 \end{aligned}$$

### Sphere

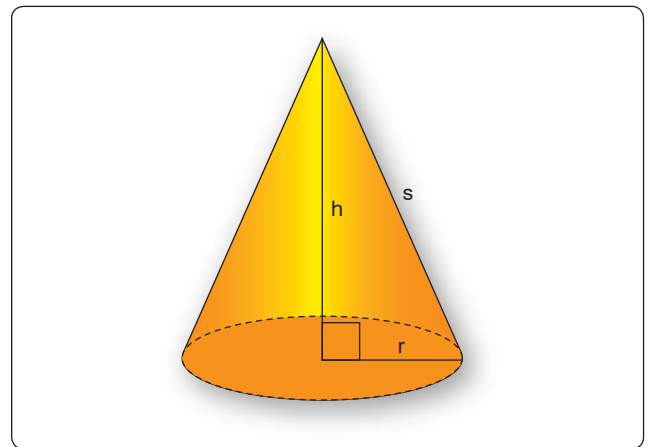
The formula for the surface area of a sphere [Figure 3-28] is given as:

$$\text{Surface area} = 4 \times \pi \times \text{radius}^2 = 4 \times \pi \times r^2$$

### Cone

The formula for the surface area of a right circular cone [Figure 3-29] is given as:

$$\begin{aligned}
 \text{Surface area} &= \pi \times \text{radius} \times [\text{radius} + (\text{radius}^2 + \text{height}^2)^{1/2}] \\
 &= \pi \times r \times [r + (r^2 + h^2)^{1/2}]
 \end{aligned}$$



**Figure 3-29.** Cone.

Solid	Volume	Surface Area	Figure
Rectangle Solid	$l \times w \times h$	$2 \times [(w \times l) + (w \times h) + (l \times h)]$	3-24
Cube	$s^3$	$6 \times s^2$	3-25
Cylinder	$\pi \times r^2 \times h$	$2 \times \pi \times r^2 + \pi \times d \times h$	3-26
Sphere	$4/3 \times \pi \times r^3$	$4 \times \pi \times r^2$	3-28
Cone	$1/3 \times \pi \times r^2 \times h$	$\pi \times r \times [r + (r^2 + h^2)^{1/2}]$	3-29

Figure 3-30. Formulas to compute volume and surface area.

Figure 3-30 summarizes the formulas for computing the volume and surface area of three-dimensional solids.

## Trigonometric Functions

Trigonometry is the study of the relationship between the angles and sides of a triangle. The word trigonometry comes from the Greek trigonon, which means three angles, and metro, which means measure.

### Right Triangle, Sides, and Angles

In Figure 3-31, notice that each angle is labeled with a capital letter. Across from each angle is a corresponding side, each labeled with a lower case letter. This triangle is a right triangle because angle C is a  $90^\circ$  angle. Side “a” is opposite from angle A and is sometimes referred to as the “opposite side.” Side “b” is next to, or adjacent to, angle A and is therefore referred to as the “adjacent side.” Side “c” is always across from the right angle and is referred to as the “hypotenuse.”

### Sine, Cosine, and Tangent

The three primary trigonometric functions and their abbreviations are: sine (sin), cosine (cos), and tangent (tan). These three functions can be found on most scientific calculators. The three trigonometric functions are actually ratios comparing two of the sides of the triangle as follows:

$$\text{Sine (sin) of angle A} = \frac{\text{opposite side (side a)}}{\text{hypotenuse (side c)}}$$

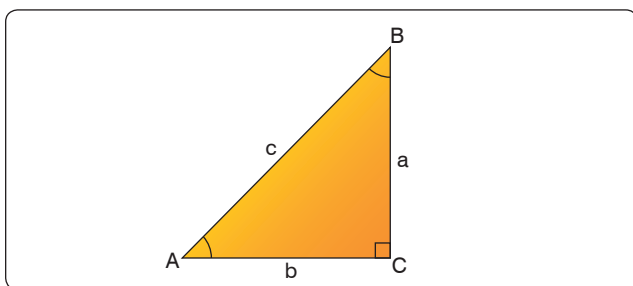


Figure 3-31. Right triangle.

$$\text{Cosine (cos) of angle A} = \frac{\text{adjacent side (side b)}}{\text{hypotenuse (side c)}}$$

$$\text{Tangent (tan) of angle A} = \frac{\text{opposite side (side a)}}{\text{adjacent side (side b)}}$$

Example: Find the sine of a  $30^\circ$  angle.

### Calculator Method:

Using a calculator, select the “sin” feature, enter the number 30, and press “enter.” The calculator should display the answer as 0.5. This means that when angle A equals  $30^\circ$ , then the ratio of the opposite side (a) to the hypotenuse (c) equals 0.5 to 1, so the hypotenuse is twice as long as the opposite side for a  $30^\circ$  angle. Therefore,  $\sin 30^\circ = 0.5$ .

### Trigonometry Table Method:

When using a trigonometry table, find  $30^\circ$  in the first column. Next, find the value for  $\sin 30^\circ$  under the second column marked “sine” or “sin.” The value for  $\sin 30^\circ$  should be 0.5.

### Pythagorean Theorem

The Pythagorean Theorem is named after the ancient Greek mathematician, Pythagoras (~500 B.C.). This theorem is used to find the third side of any right triangle when two sides are known. The Pythagorean Theorem states that  $a^2 + b^2 = c^2$ . [Figure 3-32] Where “c” = the hypotenuse of a right triangle, “a” is one side of the triangle and “b” is the other side of the triangle.

Example: What is the length of the longest side of a right triangle, given the other sides are 7 inches and 9 inches? The longest side of a right triangle is always side “c,” the hypotenuse. Use the Pythagorean Theorem to solve for the length of side “c” as follows:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + 9^2 &= c^2 \\ 49 + 81 &= c^2 \\ 130 &= c^2 \\ c &= \sqrt{130} = 11.4 \text{ inches} \end{aligned}$$

Therefore, side “c” = 11.4 inches.

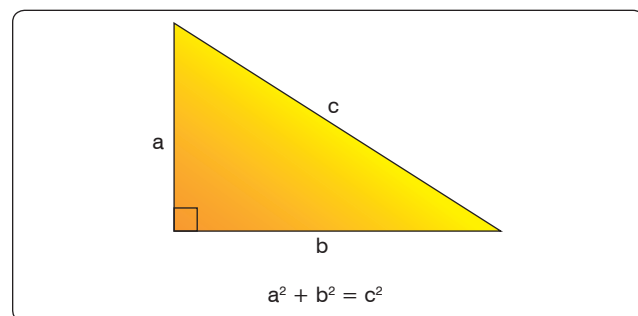


Figure 3-32. Pythagorean Theorem.

Example: The cargo door opening in a military airplane is a rectangle that is 5½ feet tall by 7 feet wide. A section of square steel plate that is 8 feet wide by 8 feet tall by 1 inch thick must fit inside the airplane. Can the square section of steel plate fit through the cargo door? It is obvious that the square steel plate will not fit horizontally through the cargo door. The steel plate is 8 feet wide and the cargo door is only 7 feet wide. However, if the steel plate is tilted diagonally, will it fit through the cargo door opening?

The diagonal distance across the cargo door opening can be calculated using the Pythagorean Theorem where “a” is the cargo door width, “b” is the cargo door height, and “c” is the diagonal distance across the cargo door opening.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (7 \text{ ft})^2 + (5.5 \text{ ft})^2 &= c^2 \\ 49 + 30.25 &= c^2 \\ 79.25 &= c^2 \\ c &= 8.9 \text{ ft} \end{aligned}$$

The diagonal distance across the cargo door opening is 8.9 feet, so the 8-foot wide square steel plate fits diagonally through the cargo door opening and into the airplane.

## Measurement Systems

### Conventional (U.S. or English) System

Our conventional (U.S. or English) system of measurement is part of our cultural heritage from the days when the thirteen colonies were under British rule. It started as a collection of Anglo-Saxon, Roman, and Norman-French weights and measures. For example, the inch represents the width of the thumb and the foot is from the length of the human foot.

Tradition holds that King Henry I decreed that the yard should be the distance from the tip of his nose to the end of his thumb. Since medieval times, commissions appointed by various English monarchs have reduced the chaos of measurement by setting specific standards for some of the most important units. Some of the conventional units of measure are: inches, feet, yards, miles, ounces, pints, gallons, and pounds. Because the conventional system was not set up systematically, it contains a random collection of conversions. For example, 1 mile = 5,280 feet and 1 foot = 12 inches.

### Metric System

The metric system, also known as the International System of Units (SI), is the dominant language of measurement used today. Its standardization and decimal features make it well-suited for engineering and aviation work.

The metric system was first envisioned by Gabriel Mouton, Vicar of St. Paul’s Church in Lyons, France. The meter is the unit of length in the metric system, and it is equal to one ten-millionth of the distance from the equator to the North Pole. The liter is the unit of volume and is equal to one cubic decimeter. The gram is the unit of mass and is equal to one cubic centimeter of water.

All of the metric units follow a consistent naming scheme, which consists of attaching a prefix to the unit. For example, since kilo stands for 1,000, one kilometer equals 1,000 meters. Centi is the prefix for one hundredth, so one meter equals one hundred centimeters. Milli is the prefix for one thousandths and one gram equals one thousand milligrams. Refer to *Figure 3-33* for the names and definitions of metric prefixes.

Symbol	Prefix	Multiplier (Exponential)	Multiplier (Numerical)	Meaning
<i>Greater Than 1</i>				
E	exa	(10 <sup>18</sup> )	1,000,000,000,000,000,000	quintillion
P	peta	(10 <sup>15</sup> )	1,000,000,000,000,000	quadrillion
T	tera	(10 <sup>12</sup> )	1,000,000,000,000	trillion
G	giga	(10 <sup>9</sup> )	1,000,000,000	billion
M	mega	(10 <sup>6</sup> )	1,000,000	million
k	kilo	(10 <sup>3</sup> )	1,000	thousand
h	hecto	(10 <sup>2</sup> )	100	hundred
da	deca	(10 <sup>1</sup> )	10	ten
<i>Less Than 1</i>				
1	unit			
d	deci	(10 <sup>-1</sup> )	0.1	tenth
c	centi	(10 <sup>-2</sup> )	0.01	hundredth
m	milli	(10 <sup>-3</sup> )	0.001	thousandth
μ	micro	(10 <sup>-6</sup> )	0.000,001	millionth
n	nano	(10 <sup>-9</sup> )	0.000,000,001	billionth
p	pico	(10 <sup>-12</sup> )	0.000,000,000,001	trillionth
f	femto	(10 <sup>-15</sup> )	0.000,000,000,000,001	quadrillionth
a	atto	(10 <sup>-18</sup> )	0.000,000,000,000,000,001	quintillionth

Figure 3-33. Names and definitions of metric prefixes.

## Measurement Systems and Conversions

The United States primarily uses the conventional (U.S. or English) system, although it is slowly integrating the metric system (SI). A recommendation to transition to the metric system within ten years was initiated in the 1970s. However, this movement lost momentum, and the United States continues to use both measurement systems. Therefore, information to convert between the conventional (U.S. or English) system and the metric (SI) system has been included in *Figure 3-23*, Applied Mathematics Formula Sheet. Examples of its use are as follows:

To convert inches to millimeters, multiply the number of inches by 25.4.

Example: 20 inches =  $20 \times 25.4 = 508$  mm

To convert ounces to grams, multiply the number of ounces by 28.35.

Example: 12 ounces =  $12 \times 28.35 = 340.2$  grams

## The Binary Number System

The binary number system has only two digits: 0 and 1. The prefix in the word “binary” is a Latin root for the word “two” and its use was first published in the late 1700s. The use of the binary number system is based on the fact that switches or valves have two states: open or closed (on/off).

Currently, one of the primary uses of the binary number system is in computer applications. Information is stored as a series of 0s and 1s, forming strings of binary numbers. An early electronic computer, ENIAC (Electronic Numerical Integrator and Calculator), was built in 1946 at the University of Pennsylvania and contained 17,000 vacuum tubes, along with 70,000 resistors, 10,000 capacitors, 1,500 relays, 6,000 manual switches and 5 million soldered joints. Computers obviously have changed a great deal since then, but are still based on the same binary number system. The binary number system is also useful when working with digital electronics because the two basic conditions of electricity, on and off, can be represented by the two digits of the binary number system. When the system is on, it is represented by the digit 1, and when it is off, it is represented by the digit 0.

## Place Values

The binary number system is a Base-2 system. That is, the place values in the binary number system are based on powers of 2. An 8-bit binary number system is shown in *Figure 3-34*.

### Converting Binary Numbers to Decimal Numbers

To convert a binary number to a decimal number, add up the place values that have a 1 (place values that have a zero do not contribute to the decimal number conversion).

Example: Convert the binary number 10110011 to a decimal number. Using the place value chart in *Figure 3-35*, add up the place values of the ‘1s’ in the binary number (ignore the place values with a zero in the binary number).

The binary number 10110011  
 $= 128 + 0 + 32 + 16 + 0 + 0 + 2 + 1$   
 $= 179$  in the decimal number system

### Converting Decimal Numbers to Binary Numbers

To convert a decimal number to a binary number, the place values in the binary system are used to create a sum of numbers that equal the value of the decimal number being converted. Start with the largest binary place value and subtract from the decimal number. Continue this process until all of the binary digits are determined.

Example: Convert the decimal number 233 to a binary number.

Start by subtracting 128 (the largest place value from the 8-bit binary number) from 233.

$233 - 128 = 105$       A “1” is placed in the first binary digit space: 1XXXXXXX.

Continue the process of subtracting the binary number place values:

$105 - 64 = 41$       A “1” is placed in the second binary digit space: 11XXXXXX.

$41 - 32 = 9$       A “1” is placed in the third binary digit space: 111XXXXX.

	Place Value								
	$2^7$ or 128	$2^6$ or 64	$2^5$ or 32	$2^4$ or 16	$2^3$ or 8	$2^2$ or 4	$2^1$ or 2	$2^0$ or 1	
10011001 shown as	1	0	0	1	1	0	0	1	=153
00101011 shown as	0	0	1	0	1	0	1	1	=43

Figure 3-34. Binary system.

Place Value																
	$2^7$ or 128	$2^6$ or 64	$2^5$ or 32	$2^4$ or 16	$2^3$ or 8	$2^2$ or 4	$2^1$ or 2	$2^0$ or 1								
10110011 shown as	1	0	1	1	0	0	1	1								
	128	+	0	+	32	+	16	+	0	+	0	+	2	+	1	=179

**Figure 3-35.** Place value chart.

Since 9 is less than 16 (the next binary place value), a “0” is placed in the fourth binary digit space, 1110XXXX.

$9 - 8 = 1$                       A “1” is placed in the fifth binary digit space: 11101XXX

Since 1 is less than 4 (the next binary place value), a 0 is placed in the sixth binary digit space: 111010XX.

Since 1 is less than 2 (the next binary place value), a 0 is placed in the seventh binary digit space: 1110100X.

$1 - 1 = 0$                       A “1” is placed in the eighth binary digit space: 11101001.

The decimal number 233 is equivalent to the binary number 11101001, as shown in *Figure 3-36*.

Three additional decimal number to binary number conversions are shown in *Figure 3-36*.

Place Value								
	$2^7$ or 128	$2^6$ or 64	$2^5$ or 32	$2^4$ or 16	$2^3$ or 8	$2^2$ or 4	$2^1$ or 2	$2^0$ or 1
35 shown as	0	0	1	0	0	0	1	1
124 shown as	0	1	1	1	1	1	0	0
96 shown as	0	1	1	0	0	0	0	0
255 shown as	1	1	1	1	1	1	1	1
233 shown as	1	1	1	0	1	0	0	1

**Figure 3-36.** Conversion from decimal number to binary number.

